Selective Revision by Deductive Argumentation

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Abstract. The *success* postulate of classic belief revision theory demands that after revising some beliefs with by information the new information is believed. However, this form of *prioritized* belief revision is not apt under many circumstances. Research in *non-prioritized* belief revision investigates forms of belief revision where *success* is not a desirable property. Herein, *selective revision* uses a two step approach, first applying a transformation function to decide if and which part of the new information shall be accepted, and second, incorporating the result using a prioritized revision operator. In this paper, we implement a transformation function by employing deductive argumentation to assess the value of new information. Hereby we obtain a non-prioritized revision operator that only accepts new information if believing in the information is justifiable with respect to the beliefs. By making use of previous results on selective revision we prove that our revision operator satisfies several desirable properties. We illustrate the use of the revision operator by means of examples and compare it with related work.

1 Introduction

Belief revision [4, 12] is concerned with changing beliefs in the light of new information. Usually, the beliefs of an agent are not static but change when new information is available. In order to be able to act reasonably in a changing environment the agent has to integrate new information and give up outdated beliefs. In particular, if the agent learns that some beliefs have been misleadingly assumed to be true its beliefs have to be *revised*. The research field of belief revision distinguishes between *prioritized* and *non-prioritized* belief revision. In prioritized belief revision [12] new information is always assumed to represent the most reliable and correct information available and revising the agent's beliefs by the new information is expected to result in believing the new information. This is a reasonable assumption for many imaginable situations and there are many technical challenges in realizing prioritized belief revision, cf. e. g. [12]. However, many circumstances demand that new information is not blindly accepted but weighted against the current beliefs. The field of non-prioritized belief revision [11] investigates change operations where revising some beliefs by new information may not result in believing the new information. Imagine a multi-agent system where agents exchange information. In general, agents may be cooperative or competitive. Information that is passed from one agent to another may be intentionally wrong, mistakenly wrong, or correct. It is up to the receiver of the information to evaluate whether it should be integrated into the beliefs or not. In particular, in non-prioritized belief revision the satisfaction of the *success* postulate—which demands that new information is believed after revision—is not desirable. In [9] a specific class of non-prioritized belief revision operators is investigated. A *selective revision* is a two-step revision that consists of 1.) filtering new information using a *transformation function* and 2.) revising the beliefs with the result of the filtering in a prioritized way. In [9], no concrete implementations of the transformation function are given but several results are proven that show how specific properties for the transformation function and the *inner* prioritized revision translate to specific properties for the *outer* non-prioritized revision.

In this paper we propose a specific implementation of a transformation function that makes use of *deductive argumentation* [2]. A deductive argumentation theory is a set of propositional sentences and an argument for some sentence ϕ is a minimal proof for ϕ. If the theory is inconsistent there may also be proofs for the complement of a sentence $\neg\phi$ and in order to decide whether ϕ or $\neg\phi$ is to be believed, an argumentative evaluation is performed that compares arguments with counterarguments. We use the framework of [2] to implement a transformation function for selective revision that decides for each individual piece of information whether to accept it for revision or not, based on its argumentative evaluation. In particular, we consider the case that revision is to be performed based on a set of pieces of information instead of just a single piece of information. By doing so, we allow new information to contain arguments. As a result, an agent decides whether to accept some new information on the basis of its own evaluation of the information and the arguments that may be contained in this information. Consider the following example.

Example 1. Imagine the agent *Anna* wants to spend her holidays on Hawaii. She is aware of the fact that there has been some volcano activity on Hawaii recently but is convinced there is no immediate danger. Anna's boss *Bob* doesn't want Anna to go on vacation at this time of the year and tells her that she has to do some work here and should not go to Hawaii. However, Anna wants to go surfing and this is a much better argument for her to go to Hawaii instead of staying at work. As a consequence she rejects Bob's argument to stay and does not revise her beliefs. Consider now that *Carl*, a good friend of *Anna*, is a vulcanologist and tells Anna that there is actually an immediate danger of an eruption. Anna knows that Carl is clearly knowledgable about volcanos and finds his argument convincing. Consequently, she accepts the new information and revises her beliefs accordingly. ■

In the previous example the decisions of the agent Anna resulted in either accepting or rejecting the new information completely. However, it may also be the case that some of the new information is accepted and some is rejected. Consider the following example.

Example 2. Imagine Bob tells Anna that she has to stay for work because all her colleagues are having a vacation at the same time and she has to fill in for them. Suppose Anna knows that there is no work to do during her planned vacation as all clients of her company are on vacation as well. Then Anna would reject the conclusion of Bob's argument that she has to stay, but might very well accept that all her colleagues will be on vacation as well. ■

In this paper we develop an approach for selective revision that is capable of deciding whether to accept, reject, or partially accept some new information, based on deductive argumentation. In order to do so we also extend the notions of selective revision to the problem of multiple base revision, i. e., the problem of revising a belief base (instead of a belief set) by a set of sentences.

The rest of this paper is organized as follows. In Section 2 we introduce some necessary technical preliminaries. We go on in Section 3 with providing an overview on the notions of belief revision and extending the approach of selective revision to selective multiple base revision. We continue in Section 4 with presenting the framework of deductive argumentation. In Section 5 we propose our implementation of selective multiple base revision via deductive argumentation and investigate its properties. In Section 6 we review some related work and in Section 7 we conclude.

2 Preliminaries

In this paper we suppose that the beliefs of an agent are given in the form of propositional sentences. Let At be a propositional signature, i. e. a set of propositional atoms. Let $\mathcal{L}(\mathsf{At})$ be the corresponding propositional language generated by the atoms in At and the connectives $\wedge$ (*and*), $\vee$ (*or*), $\Rightarrow$ (*implication*), and $\neg$ (*negation*). As a notational convenience we assume some arbitrary total order $\succ$ on the elements of $\mathcal{L}(\mathsf{At})$ which is used to enumerate elements of each finite $\Phi \subseteq \mathcal{L}(\mathsf{At})$ in a unique way, cf. [2]. For a finite subset $\Phi \subseteq \mathcal{L}(\mathsf{At})$ the *canonical enumeration* of Φ is the vector $\langle \phi_1, \ldots, \phi_n \rangle$ such that $\{\phi_1, \ldots, \phi_n\} = \Phi$ and $\phi_i \succ \phi_j$ for every $i < j$ with $i, j = 1, \ldots, n$. As $\succ$ is total the canonical enumeration of every finite subset $\Phi \subseteq \mathcal{L}(\mathsf{At})$ is uniquely defined.

We use the operator $\vdash$ to denote classical entailment, i. e., for sets of propositional sentences $\Phi_1, \Phi_2 \subseteq \mathcal{L}(\mathsf{At})$ we say that Φ_2 *follows* from Φ_1, denoted by $\Phi_1 \vdash \Phi_2$, if and only if Φ_2 is entailed by Φ_1 in the classical logical sense. For sentences $\phi, \phi' \in \mathcal{L}(\mathsf{At})$ we write $\phi \vdash \phi'$ instead of $\{\phi\} \vdash \{\phi'\}$. We define the deductive closure $Cn(\cdot)$ of a set of sentences Φ as $Cn(\Phi) = \{\phi \in \mathcal{L}(\mathsf{At}) \mid \Phi \vdash \phi\}$. Two sets of sentences $\Phi, \Phi' \subseteq \mathcal{L}(\mathsf{At})$ are *equivalent*, denoted by $\Phi \equiv^p \Phi'$, if and only if it holds that $\Phi \vdash \Phi'$ and $\Phi' \vdash \Phi$. We also use the equivalence relation $\cong^p$ which is defined as $\Phi \cong^p \Phi'$ if and only if there is a bijection $\sigma : \Phi \to \Phi'$ such that for every $\phi \in \Phi$ it holds that $\phi \equiv^p \sigma(\phi)$. This means that $\Phi \cong^p \Phi'$ if Φ and Φ' are *element-wise* equivalent. Note that $\Phi \cong^p \Phi'$ implies $\Phi \equiv^p \Phi'$ but not vice versa. In particular, it holds that e. g $\{a \wedge b\} \equiv^p \{a, b\}$ but $\{a \wedge b\} \not\cong^p \{a, b\}$. For sentences $\phi, \phi' \in \mathcal{L}(\mathsf{At})$ we write $\phi \equiv \phi'$ instead of $\{\phi\} \equiv \{\phi'\}$ if $\equiv \in \{\equiv^p, \cong^p\}$. If $\Phi \vdash \perp$ we say that Φ is *inconsistent*.

For a set S let $\mathfrak{P}(S)$ denote the power set of S, i. e. the set of all subsets of S. For a set S let $\mathfrak{PP}(S)$ denote the set of multi-sets of S, i. e. the set of all subsets of S where an element may occur more than once. To distinguish sets from multi-sets we use brackets "$\langle$" and "$\rangle$" for the latter.

3 Selective Multiple Base Revision

The field of belief revision is concerned with the change of beliefs when more recent or more reliable information is at hand. The most important description of properties of *prioritized* belief change operators is given by Alchourrón, Gärdenfors and Makinson in their seminal paper [4]. The usual framework for representing beliefs considered for belief revision is that of *belief sets* which are revised by a single sentence. A *belief set* S is a subset of $\mathcal{L}(\mathsf{At})$ that is deductively closed, i. e., $S = Cn(S)$. Working with belief sets in practice is unmanageable due to their infinite size. The more practical representation form are *belief bases* which are finite sets of sentences. These also come with the advantage of making it possible to differentiate between explicit and inferred beliefs, cf. [12]. In this work we consider the problem of *multiple base revision*. That is, we employ belief bases for knowledge representation and we consider revising a belief base by a set of sentences, cf. the notion of *multiple revision* in [12].

Let $\mathcal{K} \subseteq \mathcal{L}(\mathsf{At})$ be a belief base, $\Phi \subseteq \mathcal{L}(\mathsf{At})$ be some set of sentences, and consider the problem of changing $\mathcal{K}$ in order to entail Φ. If $\mathcal{K} \cup \Phi$ is consistent then there is no need for contracting the existing beliefs and the problem can be solved via *expansion* $\mathcal{K} + \Phi$ which is characterized via $\mathcal{K} \cup \Phi$. If $\mathcal{K} \cup \Phi$ is inconsistent, conflicts arising from the addition of Φ to $\mathcal{K}$ have to be resolved. In general, this means that some of the current beliefs have to be given up in order to come up with a consistent belief base. The AGM framework [4] proposes several basic postulates a revision operator should obey. As we consider belief bases for knowledge representation we start with the corresponding postulates for belief base revision [12] adapted to revision by sets of sentences [8]. Let $*$ be a multiple base revision operator—i. e., if $\mathcal{K}$ and Φ are sets of sentences so is $\mathcal{K} * \Phi$—and consider the following postulates.

Success. $\mathcal{K} * \Phi \vdash \Phi$.
Inclusion. $\mathcal{K} * \Phi \subseteq \mathcal{K} + \Phi$.
Vacuity. If $\mathcal{K} \cup \Phi \not\vdash \perp$ then $\mathcal{K} + \Phi \subseteq \mathcal{K} * \Phi$.
Consistency. If Φ is consistent then $\mathcal{K} * \Phi$ is consistent.
Relevance. If $\alpha \in (\mathcal{K} \cup \Phi) \setminus (K * \Phi)$ then there is a set H such that $\mathcal{K} * \Phi \subseteq H \subseteq K \cup \Phi$ and H is consistent but $H \cup \{\alpha\}$ is inconsistent.

Another important property for the framework of [4] is *extensionality* which can be phrased for multiple base revision as follows.

Extensionality. If $\Phi \equiv^p \Psi$, then $\mathcal{K} * \Phi \equiv^p \mathcal{K} * \Psi$.

The above property is usually not considered for the problem of base revision as base revision is motivated by observing syntax and not (only) semantic contents. In particular, for the problem of multiple base revision, satisfaction of *extensionality* imposes that $\mathcal{K} * \{a, b\} \equiv^p \mathcal{K} * \{a \wedge b\}$ as $\{a, b\} \equiv^p \{a \wedge b\}$. Identifying the "comma"-operator with the logical "AND"-operator is not always a reasonable thing to do, see e. g. [5] for a discussion. However, we consider the following weakened form of *extensionality*.

Weak Extensionality. If $\Phi \cong^p \Phi'$ then $\mathcal{K} * \Phi \equiv^p \mathcal{K} * \Phi'$.

The property *weak extensionality* only demands that the outcomes of the revisions $\mathcal{K} * \Phi$ and $\mathcal{K} * \Phi'$ are equivalent if Φ and Φ' are element-wise equivalent.

Definition 1. *A revision operator $*$ is called a* prioritized multiple base revision operator *if $*$ satisfies* success*,* inclusion*,* vacuity*,* consistency*,* relevance*, and* weak extensionality*.*

For non-prioritized multiple base revision the properties *inclusion*, *vacuity*, *consistency*, *relevance*, and *weak extensionality* can also be regarded as desirable. This is not the case for *success* is general but we can replace *success* by weakened versions, cf. [11]. We denote with $\circ$ a non-prioritized belief revision operator, i. e., $\mathcal{K} \circ \Phi$ is the non-prioritized revision of $\mathcal{K}$ by Φ. Then consider the following properties for $\circ$, cf. [9].

Weak Success. If $\mathcal{K} \cup \Phi \nvdash \perp$ then $\mathcal{K} \circ \Phi \vdash \Phi$.
Consistent Expansion. If $\mathcal{K} \not\subseteq \mathcal{K} \circ \Phi$ then $\mathcal{K} \cup (\mathcal{K} \circ \Phi) \vdash \perp$.

Note that *weak success* follows from *vacuity*, and *consistent expansion* follows from *vacuity* and *success*, cf. [9].

Definition 2. *A revision operator $\circ$ is called* non-prioritized multiple base revision operator *if $\circ$ satisfies* inclusion*,* consistency*,* weak extensionality*,* weak success*, and* consistent expansion*.*

We do not require *relevance* to be satisfied by non-prioritized multiple base revisions as it is hardly achievable in the context of selective revision, see below. For the following, bear in mind that the main difference between a prioritized multiple base revision operator $*$ and a non-prioritized multiple base revision operator $\circ$ is that $\mathcal{K} * \Phi \vdash \Phi$ is required but $\mathcal{K} \circ \Phi \vdash \Phi$ is not.

A specific approach to non-prioritized belief revision is *selective revision* [9]. There, the problem of revising a belief set S with a single sentence α is realized by applying a *transformation function* f to α, obtaining a new sentence α', and then revising S by α' in a prioritized way. The transformation function f is supposed to determine whether α should be accepted as a whole or whether it should be somewhat weakened. We adopt the notions of [9] for the problem of *selective multiple belief base revision* and still consider the problem of revising a belief base $\mathcal{K}$ by some set Φ of sentences. Following the ideas of [9] we define the selective multiple base revision $\mathcal{K} \circ \Phi$ via

$$\mathcal{K} \circ \Phi = \mathcal{K} * f_{\mathcal{K}}(\Phi) \tag{1}$$

with a transformation function $f_{\mathcal{K}} : \mathfrak{P}(\mathcal{L}(\mathsf{At})) \rightarrow \mathfrak{P}(\mathcal{L}(\mathsf{At}))$ and some (prioritized) multiple base revision $*$. In [9] several properties for transformation functions in the context of belief set revision are discussed. We rephrase some of them here slightly to fit the framework of multiple base revision. Let $\mathcal{K} \subseteq \mathcal{L}(\mathsf{At})$ be consistent and let $\Phi, \Phi' \subseteq \mathcal{L}(\mathsf{At})$.

Inclusion. $f_{\mathcal{K}}(\Phi) \subseteq \Phi$
Weak Inclusion. If $\mathcal{K} \cup \Phi$ is consistent then $f_{\mathcal{K}}(\Phi) \subseteq \Phi$
Extensionality. If $\Phi \equiv^p \Phi'$ then $f_{\mathcal{K}}(\Phi) \equiv^p f_{\mathcal{K}}(\Phi')$
Consistency Preservation. If Φ is consistent then $f_{\mathcal{K}}(\Phi)$ is consistent
Consistency. $f_{\mathcal{K}}(\Phi)$ is consistent
Maximality. $f_{\mathcal{K}}(\Phi) = \Phi$
Weak Maximality. If $\mathcal{K} \cup \Phi$ is consistent then $f_{\mathcal{K}}(\Phi) = \Phi$

We also consider the following novel property.

Weak Extensionality. If $\Phi \cong^p \Phi'$ then $f_{\mathcal{K}}(\Phi) \cong^p f_{\mathcal{K}}(\Phi')$

Not all of the above properties may be desirable for a transformation function that is to be used for selective revision. For example, the property *maximality* states that $f_{\mathcal{K}}$ should not modify the set Φ. Satisfaction of this property makes (1) equivalent to $\mathcal{K} * \Phi$. As $*$ is meant to be a prioritized revision function we lose the possibility for non-prioritized revision.

Note that for *weak extensionality* we demand $f_{\mathcal{K}}(\Phi)$ and $f_{\mathcal{K}}(\Phi')$ to be element-wise equivalent instead of just equivalent (in contrast to the property *weak extensionality* for revision). We do this because $f_{\mathcal{K}}$ is supposed to be applied in the context of base revision which is sensitive to syntactic variants. We introduce the postulate *weak extensionality* for transformation functions with the same motivation as we do for multiple base revision. However, for the case of transformation functions the problem with satisfaction of *extensionality* is more apparent. Consider again $\Phi = \{a, b\}$ and $\Phi' = \{a \wedge b\}$. It follows that $\Phi \equiv^p \Phi'$ and if $f_{\mathcal{K}}$ satisfies *extensionality* this results in $f_{\mathcal{K}}(\{a, b\}) \equiv^p f_{\mathcal{K}}(\{a \wedge b\})$. If $f_{\mathcal{K}}$ also satisfies *inclusion* it follows that $f_{\mathcal{K}}(\{a \wedge b\}) \in \{\emptyset, \{a \wedge b\}\}$ and therefore $f_{\mathcal{K}}(\{a, b\}) \in \{\emptyset, \{a, b\}\}$. In general, if $f_{\mathcal{K}}$ satisfies both *inclusion* and *extensionality* it follows that either $f_{\mathcal{K}}(\Phi) = \emptyset$ or $f_{\mathcal{K}}(\Phi) = \Phi$ for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$ (as Φ is equivalent to a Φ' that consists of a single formula that is the conjunction of the formulas in Φ and $f_{\mathcal{K}}(\Phi') = \emptyset$ or $f_{\mathcal{K}}(\Phi') = \Phi'$ due to *inclusion*). As we are interested in a more graded approach to belief revision we want to be able to accept or reject specific pieces of Φ and not just Φ as a whole. Consequently, we consider *weak extensionality* as a desirable property instead of *extensionality*. Note that *extensionality* implies *weak extensionality* as $\Phi \cong^p \Phi'$ implies $\Phi \equiv^p \Phi'$.

In [9] several representation theorems are given that characterize non-prioritized belief revision by selective revision via (1) and specific properties of $*$

and $f_{\mathcal{K}}$. In particular, it is shown that a reasonable non-prioritized belief revision operator $\circ$ can be characterized by an AGM revision $*$ and a transformation function $f_{\mathcal{K}}$ that satisfies *extensionality*, *consistency preservation*, and *weak maximality*. Note, however, that [9] deals with the problem of revising a belief set by a single sentence. Nonetheless, we can carry over the results of [9] to the problem of multiple base revision and obtain the following result.

Proposition 1. *Let $*$ be a prioritized multiple base revision operator and let $f_{\mathcal{K}}$ satisfy* inclusion*,* weak extensionality*,* consistency preservation*, and* weak maximality*. Then $\circ$ defined via (1) is a non-prioritized multiple base revision operator.*

Proof. We have to show that $\circ$ satisfies inclusion*,* consistency*,* weak extensionality*,* weak success*, and* consistent expansion*.*

- Inclusion*. It holds that $f_{\mathcal{K}}(\Phi) \subseteq \Phi$ as $f_{\mathcal{K}}$ satisfies* inclusion*. Also, $*$ satisfies* inclusion *and it follows $\mathcal{K} * f_{\mathcal{K}}(\Phi) \subseteq \mathcal{K} \cup f_{\mathcal{K}}(\Phi) \subseteq \mathcal{K} \cup \Phi$.*
- Consistency*. If Φ is consistent so is $f_{\mathcal{K}}(\Phi)$ as $f_{\mathcal{K}}$ satisfies* consistency preservation*. As $*$ satisfies* consistency *it follows that $\mathcal{K} * f_{\mathcal{K}}(\Phi)$ is consistent.*
- Weak Extensionality*. If $\Phi \cong^p \Phi'$ then $f_{\mathcal{K}}(\Phi) \cong^p f_{\mathcal{K}}(\Phi')$ as $f_{\mathcal{K}}$ satisfies* weak extensionality*. It follows that $\mathcal{K} * f_{\mathcal{K}}(\Phi) \equiv^p \mathcal{K} * f_{\mathcal{K}}(\Phi')$ as $*$ satisfies* weak extensionality*.*
- Weak Success*. If $\mathcal{K} \cup \Phi$ is consistent it follows that $f_{\mathcal{K}}(\Phi) = \Phi$ as $f_{\mathcal{K}}$ satisfies* weak maximality*. As $*$ satisfies* vacuity *it follows $\mathcal{K} + \Phi \subseteq \mathcal{K} * f_{\mathcal{K}}(\Phi)$. Hence, $\circ$ satisfies vacuity as well and therefore* weak success*.*
- Consistent Expansion*. Suppose $\mathcal{K} \not\subseteq \mathcal{K} * f_{\mathcal{K}}(\Phi)$. Note that $*$ satisfies* consistent expansion *as $*$ satisfies* vacuity *and* success*, cf. [9]. It follows that $\mathcal{K} \cup \{\mathcal{K} * f_{\mathcal{K}}(\Phi)\}$ is inconsistent.* □

Note that *relevance* does not hold for $\mathcal{K} \circ \Phi$ defined via (1) in general. Consider for example the transformation function $f^0_{\mathcal{K}}$ defined via $f^0_{\mathcal{K}}(\Phi) = \Phi$ if $\mathcal{K} \cup \Phi$ is consistent and $f^0_{\mathcal{K}}(\Phi) = \emptyset$ otherwise. Then $f^0_{\mathcal{K}}$ satisfies all properties for transformation functions except *maximality*. But it is easy to see that $\mathcal{K} \circ \Phi$ defined via (1) using $f^0_{\mathcal{K}}$ and a prioritized multiple base revision operator $*$ fails to satisfy *relevance*. We leave it to future work to investigate further properties for transformation functions that may enable *relevance* to hold in general.

In the following we aim at implementing a selective multiple base revision using deductive argumentation and go on with introducing the latter.

4 Deductive Argumentation

Argumentation frameworks [1] allow for reasoning with inconsistent information based on the notions of arguments, counterarguments and their relationships. Since the seminal paper [6] interest has grown in research in computational models for argumentation that allow for a coherent procedure for consistent reasoning in the presence of inconsistency. In this paper we use the framework of *deductive argumentation* as proposed by Besnard and Hunter [2]. This framework bases on classical propositional logic and is therefore apt for our aim to use argumentation to realize a transformation function f. The central notion of the framework of deductive argumentation is that of an *argument*.

Definition 3 (Argument). *Let $\Phi \subseteq \mathcal{L}(\mathsf{At})$ be a set of sentences. An* argument *$\mathcal{A}$ for a sentence $\alpha \in \mathcal{L}(\mathsf{At})$ in Φ is a tuple $\mathcal{A} = \langle \Psi, \alpha \rangle$ with $\Psi \subseteq \Phi$ that satisfies 1.) $\Psi \not\vdash \perp$, 2.) $\Psi \vdash \alpha$, and 3.) there is no $\Psi' \subsetneq \Psi$ with $\Psi' \vdash \alpha$. For an argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ we say that α is the* claim *of $\mathcal{A}$ and Ψ is the* support *of $\mathcal{A}$.*

Hence, an argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ for α is a minimal proof for entailing α. Given a set $\Phi \subseteq \mathcal{L}(\mathsf{At})$ of sentences there may be multiple arguments for α. As in [2] we are interested in arguments that are most cautious.

Definition 4 (Conservativeness). *An argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ is more* conservative *than an argument $\mathcal{B} = \langle \Phi, \beta \rangle$ if and only if $\Psi \subseteq \Phi$ and $\beta \vdash \alpha$.*

In other words, an $\mathcal{A}$ is more conservative than an argument $\mathcal{B}$ if $\mathcal{B}$ has a smaller support (with respect to set inclusion) and a more general conclusion. An argument $\mathcal{A}$ is *strictly more conservative* than an argument $\mathcal{B}$ if and only if $\mathcal{A}$ is more conservative than $\mathcal{B}$ but $\mathcal{B}$ is not more conservative than $\mathcal{A}$. If $\Phi \subseteq \mathcal{L}(\mathsf{At})$ is inconsistent there are arguments with contradictory claims.

Definition 5 (Undercut). *An argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ is an* undercut *for an argument $\mathcal{B} = \langle \Phi, \beta \rangle$ if and only if $\alpha = \neg(\phi_1 \wedge \ldots \wedge \phi_n)$ for some $\phi_1, \ldots, \phi_n \subseteq \Phi$.*

If $\mathcal{A}$ is an undercut for $\mathcal{B}$ then we also say that $\mathcal{A}$ *attacks* $\mathcal{B}$. In order to consider only those undercuts for an argument that are most general we restrain the notion of undercut as follows.

Definition 6 (Maximally conservative undercut). *An argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ is a* maximally conservative undercut *for an argument $\mathcal{B} = \langle \Phi, \beta \rangle$ if and only if $\mathcal{A}$ is an undercut of $\mathcal{B}$ and there is no undercut $\mathcal{A}'$ for $\mathcal{B}$ that is strictly more conservative than $\mathcal{A}$.*

Definition 7 (Canonical undercut). *An argument $\mathcal{A} = \langle \Psi, \neg(\phi_1 \wedge \ldots \wedge \phi_n) \rangle$ is a* canonical undercut *for an argument $\mathcal{B} = \langle \Phi, \beta \rangle$ if and only if $\mathcal{A}$ is a maximally conservative undercut for $\mathcal{B}$ and $\langle \phi_1, \ldots, \phi_n \rangle$ is the canonical enumeration of Φ.*

It can be shown that it suffices to consider only the canonical undercuts for an argument in order to come up with a reasonable argumentative evaluation of some claim α [2]. Having an undercut $\mathcal{B}$ for an argument $\mathcal{A}$ there may also be an undercut $\mathcal{C}$ for $\mathcal{B}$ which *defends* $\mathcal{A}$. In order to give a proper evaluation of some argument $\mathcal{A}$ we have to consider all undercuts for its undercuts as well, and so on. This leads to the notion of an *argument tree*.

Definition 8 (Argument tree). *Let $\alpha \in \mathcal{L}(\mathsf{At})$ be some sentence and let $\Phi \subseteq \mathcal{L}(\mathsf{At})$ be a set of sentences. An* argument tree *$\tau_\Phi(\alpha)$ for α in Φ is a tree where the nodes are arguments and that satisfies*

1. *the root is an argument for α in Φ,*
2. *for every path $[\langle\Phi_1,\alpha_1\rangle,\ldots,\langle\Phi_n,\alpha_n\rangle]$ in $\tau_\Phi(\alpha)$ it holds that $\Phi_n \nsubseteq \Phi_1 \cup \ldots \cup \Phi_{n-1}$, and*
3. *the children $\mathcal{B}_1,\ldots,\mathcal{B}_m$ of a node $\mathcal{A}$ consist of all canonical undercuts for $\mathcal{A}$ that obey 2.).*

Let $\mathcal{T}(\mathsf{At})$ be the set of all argument trees.

An argument tree is a concise representation of the relationships between different arguments that favor or reject some argument $\mathcal{A}$. In order to evaluate whether a claim α can be justified we have to consider all argument trees for α and all argument trees for $\neg\alpha$. For an argument tree τ let $\mathsf{root}(\tau)$ denote the root node of τ. Furthermore, for a node $\mathcal{A} \in \tau$ let $\mathsf{ch}_\tau(\mathcal{A})$ denote the children of $\mathcal{A}$ in τ and $\mathsf{ch}^{\mathcal{T}}_\tau(\mathcal{A})$ denote the set of sub-trees rooted at a child of $\mathcal{A}$.

Definition 9 (Argument structure). *Let $\alpha \in \mathcal{L}(\mathsf{At})$ be some sentence and let $\Phi \subseteq \mathcal{L}(\mathsf{At})$ be a set of sentences. The* argument structure *$\Gamma_\Phi(\alpha)$ for α with respect to Φ is the tuple $\Gamma_\Phi(\alpha) = (\mathcal{P},\mathcal{C})$ such that $\mathcal{P}$ is the set of argument trees for α in Φ and $\mathcal{C}$ is the set of arguments trees for $\neg\alpha$ in Φ.*

The argument structure $\Gamma_\Phi(\alpha)$ of a $\alpha \in \mathcal{L}(\mathsf{At})$ gives a complete picture of the reasons for and against α. In order to evaluate those reasons we use the following notation, cf. [2].

Definition 10 (Categorizer). *A* categorizer *γ is a function $\gamma : \mathcal{T}(\mathsf{At}) \to \mathbb{R}$.*

A categorizer is meant to assign a value to an argument tree τ depending on how strongly this argument tree favors the root argument. In particular, the larger the value of $\gamma(\tau)$ the better the justification in believing in the claim of the root argument. For an argument structure $\Gamma_\Phi(\alpha) = (\{\tau^p_1,\ldots,\tau^p_n\},\{\tau^c_1,\ldots,\tau^c_m\})$ and a categorizer γ we abbreviate

$$\gamma(\Gamma_\Phi(\alpha)) = (\langle\gamma(\tau^p_1),\ldots,\gamma(\tau^p_n)\rangle,\langle\gamma(\tau^c_1),\ldots,\gamma(\tau^c_m)\rangle) \in \mathfrak{PP}(\mathbb{R}) \times \mathfrak{PP}(\mathbb{R})\,.$$

Definition 11 (Accumulator). *An* accumulator *κ is a function $\kappa : \mathfrak{PP}(\mathbb{R}) \times \mathfrak{PP}(\mathbb{R}) \to \mathbb{R}$.*

An accumulator is meant to evaluate the categorization of argument trees for or against some sentence α. We say that a set of sentences $\Phi \subseteq \mathcal{L}(\mathsf{At})$ *accepts* a sentence α with respect to a categorizer γ and an accumulator κ, denoted by $\Phi \mid\!\sim_{\kappa,\gamma} \alpha$ if and and only if

$$\kappa(\gamma(\Gamma_\Phi(\alpha))) > 0$$

A set of sentences $\Phi \subseteq \mathcal{L}(\mathsf{At})$ *rejects* a sentence α with respect to a categorizer γ and an accumulator κ, denoted by $\Phi \mid\!\not\sim_{\kappa,\gamma} \alpha$ if and and only if

$$\kappa(\gamma(\Gamma_\Phi(\alpha))) < 0$$

If Φ neither accepts nor rejects α with respect to γ and κ we say that Φ is *undecided* about α with respect to γ and κ. Some simple instances of categorizers and accumulators are as follows.

Example 3. Let τ be some argument tree. The classical evaluation of an argument tree—as e. g. employed in *Defeasible Logic Programming* [10]—is that each leaf of the tree is considered "undefeated" and an inner node is "undefeated" if all its children are "defeated" and "defeated" if there is at least one child that is "undefeated". This intuition can be formalized by defining the *classical categorizer* γ_0 recursively via

$$\gamma_0(\tau) = \begin{cases} 1 & \text{if } \mathsf{ch}_\tau(\mathsf{root}(\tau)) = \emptyset \\ 1 - \max\{\gamma_0(\tau') \mid \tau' \in \mathsf{ch}_\tau^{\mathcal{T}}(\mathsf{root}(\tau))\} & \text{otherwise} \end{cases}$$

Furthermore, a simple accumulator κ_0 can be defined via

$$\kappa_0(\langle N_1, \ldots, N_n\rangle, \langle M_1, \ldots, M_m\rangle) = N_1 + \ldots + N_n - M_1 - \ldots - M_m \,.$$

For example, a set of sentences $\Phi \subseteq \mathcal{L}(\mathsf{At})$ accepts a sentence α with respect to γ_0 and κ_0 if and only if there are more argument trees for α where the root argument is undefeated than argument trees for $\neg\alpha$ where the root argument is undefeated. ■

More examples of categorizers and accumulators can be found in [2]. Using those notions we are able to state for every sentence $\phi \in \Phi$ whether ϕ is accepted in Φ or not, depending on the arguments that favor α and those that reject α.

5 Selective Revision by Deductive Argumentation

Using the deductive argumentation framework presented in the previous section one is able to decide for each sentence $\alpha \in \Phi$ whether α is justifiable with respect to Φ. Note that the framework of deductive argumentation heavily depends on the actual instances of categorizer and accumulator. In the following we only consider categorizer and accumulator that comply with the following minimal requirements.

Definition 12 (Well-behaving categorizer). *A categorizer γ is called* well-behaving *if $\gamma(\tau) > \gamma(\tau')$ whenever τ consists only of one single node and τ' consists of at least two nodes.*

In other words, a categorizer γ is well-behaving if the argument tree that has no undercuts for its root is considered the best justification for the root.

Definition 13 (Well-behaving accumulator). *An accumulator κ is called* well-behaving *if and only if $\kappa((\mathcal{P}, \mathcal{C})) > 0$ whenever $\mathcal{P} \neq \emptyset$ and $\mathcal{C} = \emptyset$.*

This means, that if there are no arguments against a claim α and at least one argument for α in Φ then α should be accepted in Φ. Note that both γ_0 and κ_0 are well-behaving as well as all categorizers and accumulators considered in [2]. Note further that if Φ is consistent then every sentence $\alpha \in \Phi$ is accepted by Φ with respect to every well-behaving categorizer and well-behaving accumulator.

Let $\mathcal{K} \subseteq \mathcal{L}(\mathsf{At})$ be a consistent set of sentences, and let γ be some well-behaving categorizer and κ be some well-behaving accumulator. We consider again a selective revision $\circ$ of the form (1). In order to determine the outcome of the non-prioritized revision $\mathcal{K} \circ \Phi$ for some $\Phi \subseteq \mathcal{L}(\mathsf{At})$ we implement a transformation function f that checks for every sentence $\alpha \in \Phi$ whether α is accepted in $\mathcal{K} \cup \Phi$. Note that although $\mathcal{K}$ is consistent the union $\mathcal{K} \cup \Phi$ is not necessarily consistent which gives rise to an argumentative evaluation. In the following, we consider *two* different transformation functions based on deductive argumentation. We define the *skeptical transformation function* $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}$ via

$$\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) = \{\alpha \in \Phi \mid \mathcal{K} \cup \Phi \mid\!\sim_{\kappa,\gamma} \alpha\}$$

for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$ and the *credulous transformation function* $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}$ via

$$\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) = \{\alpha \in \Phi \mid \mathcal{K} \cup \Phi \not\mid\!\sim_{\kappa,\gamma} \neg\alpha\}$$

for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$. In other words, the value of $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi)$ consists of those sentences of Φ that are accepted in $\mathcal{K} \cup \Phi$ and the value of $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}(\Phi)$ consists of those sentences of Φ that are not rejected in $\mathcal{K} \cup \Phi$. There is a subtle difference in the behavior of those two transformation functions as the following example shows.

Example 4. Let $\mathcal{K}_1 = \{a\}$ and $\Phi_1 = \{\neg a\}$. Note that there is exactly one argument tree τ_1 for $\neg a$ and one argument tree τ_2 for a in $\mathcal{K}_1 \cup \Phi$. In τ_1 the root is the argument $\mathcal{A} = \langle\{\neg a\}, \neg a\rangle$ which has the single canonical undercut $\mathcal{B} = \langle\{a\}, a\rangle$. In τ_2 the situation is reversed and the root of τ_2 is the argument $\mathcal{B}$ which has the single canonical undercut $\mathcal{A}$. Therefore, the argument structure for $\neg a$ is given via $\Gamma_{\mathcal{K}\cup\Phi}(\neg a) = (\{\tau_1\}, \{\tau_2\})$. It follows that $\gamma_0(\tau_1) = \gamma_0(\tau_2) = 0$ and $\kappa_0(\gamma_0(\Gamma_{\mathcal{K}\cup\Phi}(a))) = \kappa_0(\langle 0, 0\rangle) = 0$. It follows that $\mathcal{K} \cup \Phi$ is undecided about both $\neg a$ and a. Consequently, it follows that

$$\mathsf{S}_{\mathcal{K}_1}^{\gamma_0,\kappa_0}(\Phi_1) = \emptyset \qquad\qquad \mathsf{C}_{\mathcal{K}_1}^{\gamma_0,\kappa_0}(\Phi_1) = \{\neg a\}\,.$$

■

Let $*$ be some (prioritized) multiple base revision operator, γ some categorizer, and κ some accumulator. Using the skeptical transformation function we can define the *skeptical argumentative revision* $\circ_S^{\gamma,\kappa}$ following (1) via

$$\mathcal{K} \circ_S^{\gamma,\kappa} \Phi = \mathcal{K} * \mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) \tag{2}$$

for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$ and using the credulous transformation function we can define the *credulous argumentative revision* $\circ_C^{\gamma,\kappa}$ via

$$\mathcal{K} \circ_C^{\gamma,\kappa} \Phi = \mathcal{K} * \mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) \tag{3}$$

for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$.

Example 5. We continue Example 4. Let $*$ be some prioritized multiple base revision. Then it follows that $\mathcal{K}_1 \circ_S^{\gamma_0,\kappa_0} \Phi_1 = \{a\}$ and $\mathcal{K}_1 \circ_C^{\gamma_0,\kappa_0} \Phi_1 = \{\neg a\}$. ■

We now investigate the formal properties of the transformation functions $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}$ and $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}$ and the resulting revision operators $\circ_{S}^{\gamma,\kappa}$ and $\circ_{C}^{\gamma,\kappa}$.

Proposition 2. *Let γ be a well-behaving categorizer and κ be a well-behaving accumulator. Then the transformation functions $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}$ and $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}$ satisfy* inclusion, weak inclusion*,* weak extensionality*,* consistency preservation *and* weak maximality*.*

Proof.

- Inclusion*. This is satisfied by definition as for $\alpha \in \mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi)$ and each $\alpha \in \mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}(\Phi)$ it follows $\alpha \in \Phi$.*
- Weak Inclusion*. This follows directly from the satisfaction of* inclusion*.*
- Weak Extensionality*. Let $\Phi \cong^p \Phi'$ and let $\sigma : \Phi \to \Phi'$ be a bijection such that for every $\phi \in \Phi$ it holds that $\phi \equiv^p \sigma(\phi)$. We extend σ to $\mathcal{K}$ via $\sigma(\psi) = \psi$ for every $\psi \in \mathcal{K}$. If $\Psi \subseteq \mathcal{K} \cup \Phi$ we abbreviate*

$$\sigma(\Psi) = \bigcup_{\psi \in \Psi} \{\sigma(\psi)\}\,.$$

Let $\langle \Psi, \phi \rangle$ be an argument for some $\phi \in \Phi$ with respect to $\mathcal{K} \cup \Phi$. Then $\langle \sigma(\Psi), \sigma(\phi) \rangle$ is an argument for $\sigma(\phi)$ in $\mathcal{K} \cup \Phi'$. It follows that if τ is an argument tree for $\langle \Psi, \phi \rangle$ in $\mathcal{K} \cup \Phi$ then τ' is an argument tree for $\langle \sigma(\Psi), \sigma(\phi) \rangle$ in $\mathcal{K} \cup \Phi'$ where τ' is obtained from τ by replacing each sentence ϕ with $\sigma(\phi)$. This generalizes also to argument structures and it follows that

$$\kappa(\gamma(\Gamma_{\mathcal{K} \cup \Phi}(\phi))) = \kappa(\gamma(\Gamma_{\mathcal{K} \cup \Phi'}(\sigma(\phi))))\,.$$

Hence, $\phi \in \mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi)$ if and only if $\sigma(\phi) \in \mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi')$ for every $\phi \in \Phi$. It follows that $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) \cong^p \mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi')$. The same is true for $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}$.
- Consistency Preservation*. Every subset of a consistent set of sentences is consistent and, due to* inclusion*, it holds that $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi), \mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) \subseteq \Phi$ with consistent Φ.*
- Weak Maximality*. If $\mathcal{K} \cup \Phi$ is consistent then for all arguments for a sentence $\alpha \in \Phi$ there do not exist any undercuts as these would have to entail the negation of some sentence of the argument for α which implies inconsistency of $\mathcal{K} \cup \Phi$. The argument structure $\Gamma_{\Phi}(\alpha) = (\mathcal{P}, \mathcal{C})$ consists of one or more single node trees $\mathcal{P}$ and $\mathcal{C} = \emptyset$. As both γ and κ are well-behaving it follows that $\kappa(\gamma(\Gamma_{\Phi}(\alpha))) > 0$ for each $\alpha \in \Phi$ and therefore $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) = \Phi$ and $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) = \Phi$.* □

In particular, note that both $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}$ and $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}$ do not satisfy either *consistency* or *maximality* in general.

Corollary 1. *Let γ be a well-behaving categorizer and κ be a well-behaving accumulator. Then both $\circ_{S}^{\gamma,\kappa}$ and $\circ_{C}^{\gamma,\kappa}$ are non-prioritized multiple base revision operators.*

Proof. This follows directly from Propositions 1 and 2. □

Example 6. We continue Examples 1 and 2 and consider $\mathsf{At} = \{c, a, q, b, r, s\}$ with the following informal interprations.

c :	Anna has financial problems
a :	Anna travels to Hawaii
q :	there is volcano activity on Hawaii
b :	Anna has a lot of money
r :	Anna is a surf fanatic
s :	Anna takes a loan

Now consider Anna's belief base $\mathcal{K}_2$ given via

$$\mathcal{K}_2 = \{r,\quad r \Rightarrow a,\quad s,\quad s \Rightarrow b,\quad b \Rightarrow a,\quad b \Rightarrow \neg c\}\,.$$

This means that Anna believes that she is a surf fanatic (r), that a surf fanatic should travel to Hawaii ($r \Rightarrow a$), that she takes a loan (s), that taking a loan means that she has a lot of money ($s \Rightarrow b$), that having a lot of money implies she should travel to Hawaii ($b \Rightarrow a$), and that having a lot of money she does not have financial problems. Note that $\mathcal{K} \vdash a$, i. e. Anna intends to got to Hawaii. Now consider the new information $\Phi_2 = \{c,\quad c \Rightarrow \neg a,\quad q,\quad q \Rightarrow \neg a\}$ stemming from communication with Anna's mother. In Φ_2 the mother of Anna tries to convince her not to travel to Hawaii. In particular, Φ_2 states that Anna has financial problems (c), that having financial problems Anna should not travel to Hawaii ($c \Rightarrow \neg a$), that there is also volcano activity on Hawaii (q), and that given volcano activity Anna should not travel to Hawaii ($q \Rightarrow \neg a$).

As one can see there a several arguments for and against a in $\mathcal{K}_2 \cup \Phi_2$, e. g., $\langle r, r \Rightarrow a, a\rangle$, $\langle c, c \Rightarrow \neg a, \neg a\rangle$. We do not go into details regarding the argumentative evaluation of the sentences in Φ_2. We only note that $\mathcal{K}_2 \cup \Phi_2$ is undecided about c but accepts $c \Rightarrow \neg a$, q, and $q \Rightarrow \neg a$ with respect to γ_0 and κ_0. Consequently, the values of $\mathsf{S}^{\gamma_0,\kappa_0}_{\mathcal{K}_2}(\Phi_2)$ and $\mathsf{C}^{\gamma_0,\kappa_0}_{\mathcal{K}_2}(\Phi_2)$ are given via

$$\mathsf{S}^{\gamma_0,\kappa_0}_{\mathcal{K}_2}(\Phi_2) = \Phi_2 \setminus \{c\} \quad \text{and} \quad \mathsf{C}^{\gamma_0,\kappa_0}_{\mathcal{K}_2}(\Phi_2) = \Phi_2\,.$$

Let $*$ be some prioritized multiple base revision operator and define $\circ^{\gamma_0,\kappa_0}_S$ and $\circ^{\gamma_0,\kappa_0}_C$ via (2) and (3), respectively. Then some possible revisions of $\mathcal{K}_2$ with Φ_2 are given via

$$\mathcal{K}_2 \circ^{\gamma_0,\kappa_0}_S \Phi_2 = \{r,\ s \Rightarrow b,\ b \Rightarrow a,\ b \Rightarrow \neg c,\ c \Rightarrow \neg a,\ q \Rightarrow \neg a,\ q\}$$
$$\mathcal{K}_2 \circ^{\gamma_0,\kappa_0}_C \Phi_2 = \{r,\ s \Rightarrow b,\ b \Rightarrow a,\ b \Rightarrow \neg c,\ c \Rightarrow \neg a,\ c,\ q \Rightarrow \neg a,\ q\}\,.$$

Note that it holds $\mathcal{K}_2 \circ^{\gamma_0,\kappa_0}_S \Phi_2 \vdash \neg a$ and $\mathcal{K}_2 \circ^{\gamma_0,\kappa_0}_C \Phi_2 \vdash \neg a$. Hence, Anna accepts the conclusion of her mother's arguments not to travel Hawaii. However, if she revises her beliefs in a skeptical way she does not accept that she has financial problems. ■

6 Related Work

In terms of related work there are mainly two areas that are related to the work presented here. On the one hand, non-prioritized belief revision and on the other hand belief revision by argumentation. In the former area we instantiate and extended the non-prioritized revision operator of selective revision presented in [9] towards multiple revision and to revision of belief bases. Selective revision is one of the most general non-prioritized revision operator of the type *decision+revision* [11]. Moreover it allows for partial acceptance of the input, in contrast to most other approaches. Apart from *decision+revision* approaches there are *expansion+consolidation* approaches to non-prioritized belief revision. These perform a simple expansion by the new information, i.e. $\mathcal{K} \cup \Phi$, and then apply a consolidation operator ! that restores consistency, i.e. $\mathcal{K} * \Phi = (\mathcal{K} \cup \Phi)!$. This approach is limited to belief bases since all inconsistent belief sets are equal, i. e. $Cn(\perp) = \mathcal{L}(\mathsf{At})$. An instantiation of such an operator that is similar to the setup used in this work has been presented in [8]. The considered input to the revision consists of a set of sentences that form an explanation of some claim in the same form as the argument definition used here. However, as with all approaches of the type *expansion+consolidation*, new and old information are completely equal to the consolidation operator. In contrast, the approach presented here which makes use of two different mechanisms to first decide about if, and which part, of the input shall be accepted just considering the new information, and then performing prioritized belief revision of the old information. Also, there are *integrated choice* approaches that do not feature a two step process but a single step process applying the same technique for the selection and revision process. Mostly these approaches need some meta information, e. g. an epistemic entrenchment relation, and thus differ on the basic process as well as on the information needed.

While there has been some work on the revision of argumentation systems, very little work on the application of argumentation techniques for the revision process has been done so far, cf. [7]. In fact, the work most related to the work presented here makes use of negotiation techniques for belief revision [3, 13], without argumentation. In the general setup of [3] a symmetric merging of information from two sources is performed by means of a negotiation procedure that determines which source has to reduce its information in each round. The information to be given up is determined by another function. The negotiation ends when a consistent union of information is reached. While this can be seen as a one step process of merging or consolidation in general, the formalism also allows to differentiate between the information given up from the first source and the second source. In [3], this setting is then successively biased towards prioritizing the second source which leads to representation theorems for operations equivalent to selective revision satisfying *consistent expansion* and for classic AGM operators. While those results are interesting, the negotiation framework used in [3] is very different from the argumentation formalism used here and also very different from the setup of selective revision. Moreover, the functions for the negotiation and concession are left abstract. In [13] mutual belief revision is considered where two agents revise their respective belief state by information of the other agent. Both agents agree in a negotiation on the information that is accepted by each agent. The revisions of the agents are split into a selection function and two iterated revision functions which leads to operators satisfying *consistent expansion*. The selection function is then a negotiation function on two sets of beliefs that represent the sets of belief that each agent is willing to accept from the other agent that might obey game theoretic principles. This setting has a very different focus as ours and also does not specify the selection function.

7 Conclusion

In this paper we combined the research strains of selective revision and deductive argumentation in order to implement non-prioritized multiple base revision operators that only revise by those portions of the new information that are justified. We only took some first steps in investigating the properties of those revision operators but were able to show that those comply with many desirable properties for non-prioritized revision. We discussed the performance of our operators by examples and briefly compared our approach to related work.

Future work includes a deeper analysis of the revisions $\circ_S^{\gamma,\kappa}$ and $\circ_C^{\gamma,\kappa}$ and a more thorough comparison with related work.

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