unpublished draft

Credibility-based Selective Revision by Deductive Argumentation in Multi-agent Systems

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ABSTRACT. In this chapter we describe recent approaches in which argumentation is applied to the process of revising an agent's beliefs. We first present an approach to selective revision with a preprocessing step based on deductive argumentation. In this approach, a non-prioritized revision operator is proposed that only accepts new information if the information is justifiable with respect to an argumentative evaluation. We integrate the developed argumentative approach into selective revision in a multi-agent scenario with information stemming from different agents with different degrees of credibility. In this context an agent has to choose carefully which information is to be accepted for revision in order to avoid believing faulty or untrustworthy information. We extend our approach of selective revision by deductive argumentation for this setting by including credibility information in the argumentative process. New information is evaluated based on the credibility of the source in combination with all arguments favoring and opposing the new information. The evaluation process determines which part of the new information is to be accepted for revision and thereupon incorporated into the belief base by an appropriate revision operator.

1 Introduction

In this chapter we consider the issue of non-prioritized revision in a multi-agent setting in which the agent that is revising its belief base can receive information from multiple informants. This chapter joins two recent works on non-prioritized operators combining selected revision and deductive argumentation [29] and on argumentative credibility-based revision in multi-agent systems [34] in order to come up with a unified view.

Belief revision [1, 26] is concerned with changing beliefs in light of new information. Usually, the beliefs of an agent are not static but change when new information is available. In order to be able to act reasonably in a changing environment the agent has to integrate new information and give up outdated beliefs. In particular, if the agent learns that some beliefs have been misleadingly assumed to be true its beliefs have to be *revised*. The research field of belief revision distinguishes between several different types of operations which can differ in the representation of beliefs, the type on input and the properties of the operation itself. The beliefs of the agent are represented as a set of sentences which can be closed under logical consequence, and thus be an infinite one, called a *belief set*, or as a finite, non closed se, called a *belief base*. We consider the latter case of belief base revision, mainly developed by Hansson [26] in this work. It comes with the advantage of computational realizability and greater cognitive realism. It moreover allows to distinguish foundational from inferred beliefs which is closer to cognitive realism, and also to argumentation theory. The input of a revision operator can be a single sentence of a logical language or a set of such. Here we consider the more general latter case in which the operator is called a *multiple operator*. They were first considered independently by Fuhrmann, Hansson, Nieder, and Rott; for an overview see [16]. A prominent property to distinguish types of revision operations is the *success* property which leads to *prioritized* revision if satisfied and to *non-prioritized* belief revision if not. In prioritized belief revision [1, 26] new information is always assumed to represent the most reliable and correct information available and revising the agent's beliefs by the new information is expected to result in believing the new information. This has become known as the *success* postulate which demands that new information is believed after revision. However, this postulate has been questioned as it seems to imply that the new information should be blindly accepted instead of being weighted against current beliefs. The field of non-prioritized belief revision [25] investigates change operations where revising some beliefs by new information may not result in believing the new information. Imagine a multi-agent system where agents exchange information. In general, agents may be cooperative or competitive. Information that is passed from one agent to another may be intentionally wrong, mistakenly wrong, or correct. It is up to the receiver of the information to evaluate whether it should be integrated into the beliefs or not. In particular, in non-prioritized belief revision the satisfaction of the *success* postulate is not desirable. In this context, a specific class of non-prioritized belief revision operators called selective revision [14] is particularly interesting. A *selective revision* is a two-step revision that consists of 1.) filtering new information using a *transforma-*

tion function and 2.) revising the beliefs with the result of the filtering in a prioritized way. In [14], no concrete implementations of the transformation function are given although several results are proven that show how specific properties for the transformation function and the *inner* prioritized revision translate to specific properties for the *outer* non-prioritized revision.

In this chapter we present a specific implementation of a transformation function for selective revision that makes use of *deductive argumentation* [4]. A deductive argumentation theory is a set of propositional sentences, and an argument consists of some sentence ϕ and a minimal proof for ϕ. If the theory is inconsistent there may also be proofs for the complement of a sentence $\neg\phi$ and in order to decide whether ϕ or $\neg\phi$ is to be believed, an argumentative evaluation is performed that compares arguments with counterarguments. We use the framework of [4] to implement a transformation function for selective revision that decides for each individual piece of information whether to accept it for revision or not, based on its argumentative evaluation. In particular, we consider the case that revision is to be performed based on a set of pieces of information instead of just a single piece of information. By doing so, we allow new information to contain arguments. A non-prioritized revision operator is then proposed that only accepts new information if the information is justifiable with respect to an argumentative evaluation. The proposed operator can accept all, part, or none of the received information. As a result, an agent decides whether to accept some new information on the basis of its own evaluation of the information and the arguments that may be contained in this information. This allows us to implement a decision procedure to find out if the *success* postulate is adequate or not.

We motivate the main ideas of this approach by a (running) example. Consider the following scenario where an agent (*Anna*) has to decide where to spend her holidays and receives new information from her mother that has to be considered.

EXAMPLE 1 *Anna is a surf fanatic* (s) *and believes that a surf fanatic should travel to Hawaii* ($s \Rightarrow h$). *Anna has taken a loan* (l), *and taking a loan means having money available* ($l \Rightarrow m$). *Having money implies she should travel to Hawaii* ($m \Rightarrow h$), *and having money also implies she does not have financial problems* ($m \Rightarrow \neg f$). *Below we show the belief base* $\mathcal{K}_1$ *with Anna's beliefs. Observe that* $\mathcal{K}_1 \vdash h$, i. e., *from* $\mathcal{K}_1$ *Anna concludes she should go to Hawaii.*

$$\mathcal{K}_1 = \{s, \quad s \Rightarrow h, \quad l, \quad l \Rightarrow m, \quad m \Rightarrow h, \quad m \Rightarrow \neg f\}\,.$$

Now consider the new information $\Phi_1 = \{f, \quad f \Rightarrow \neg h, \quad v, \quad v \Rightarrow \neg h\}$ *that Anna has received from her mother, in order to tell her not to travel to*

Hawaii. In particular, Φ_1 states that Anna has financial problems (f), that having financial problems Anna should not travel to Hawaii ($f \Rightarrow \neg h$), that there is also volcano activity on Hawaii (v), and that given volcano activity Anna should not travel to Hawaii ($v \Rightarrow \neg h$).

Observe that in the example above, $\mathcal{K}_1 \cup \Phi_1$ is inconsistent, that is, Anna can not accept all this information without withdrawing some sentences from $\mathcal{K}_1$. In this situation, Anna could reject all the new information; she could accept all the information but withdraw some of her current beliefs; or some combination. For instance, she could reject f (financial problems) because she has taken a loan, but she could accept $\{ f \Rightarrow \neg h,\ v,\ \ v \Rightarrow \neg h\}$ provided she eliminates part of her beliefs.

In order to consider a situation such as the one described in Example 1, in Section 4 we will introduce a selective revision approach based on deductive argumentation. Deductive argumentation will be used for deciding whether to accept the whole received set, to reject all, or to accept part of the set.

Then in Section 5.2 we will show how to integrate the developed argumentative approach into selective revision in a multi-agent scenario with information stemming from different agents with different degrees of credibility. Thus, an agent can choose which information is to be accepted based on the credibility of its informants. For instance, consider the following example in which an agent (*Sam*) has to revise his knowledge and can receive information from other agents he does not consider equally credible.

EXAMPLE 2 *Consider an agent* Sam (A_S) *interacts with three other agents at his work: his boss* Bob (A_B)*, his assigned client* Carl (A_C) *and his colleague* Paul (A_P)*. Regarding beliefs related to his job, Sam considers that these agents are not equally credible: for Sam the most credible is A_C, then A_B, then A_P, and finally himself (A_S).*

Now consider that Sam believes that he has no work to do ($\neg w$), and that if he has no assigned work then he can go on vacation ($\neg w \rightarrow v$). His colleague A_P has also told him that he can replace Sam at his work (r), and in such a case he can go on vacation ($r \rightarrow v$). Finally, his client has also informed Sam, that there is no work to do $\neg w$. Hence, Sam has three arguments supporting v (to go on vacation). One supported by $\neg w$ and $\neg w \rightarrow v$; the second supported by the information received from A_P: r and $r \rightarrow v$; and the third one supported by the information received from his client that he was no work to do and can therefore go on vacation.

Consider now that his boss (A_B) informs Sam that he has work to do (w), that Paul has informed him that he is ill (i) and that if Paul is ill then he has no replacement ($i \rightarrow \neg r$). Thus Sam has to revise his beliefs in order *to incorporate part or all this new information, and it may be possible that the arguments that support v will not longer exist upon revision.*

In order to consider a situation such as the one described in Example 2, we will show in Section 5.2 an approach of selective revision by deductive argumentation for this setting by including credibility information in the argumentative process. New information will be evaluated based on the credibility of the source in combination with all arguments favoring and opposing the new information. The evaluation process determines which part of the new information is to be accepted for revision and thereupon incorporated into the belief base by an appropriate revision operator.

The rest of this chapter is organized as follows. In Section 2 we introduce some necessary technical preliminaries; and we provide an overview on the notions of belief revision and extending the approach of selective revision to selective multiple base revision. We continue in Section 3 with presenting the framework of deductive argumentation. In Section 4 we propose our implementation of selective multiple base revision via deductive argumentation and investigate its properties. Section 5 presents an epistemic model based on credibility for an agent situated in a multi-agent environment. Afterwards we present our approach to argumentative credibility-based revision of epistemic models and go on with an analysis of this approach. In Section 6 we review some related work and in Section 7 we conclude.

2 Background

In this section we fix notations and recall approaches to selective revision. We go into more details on selective multiple revision on belief bases as this will provide most important postulates and techniques for the approaches to be presented here.

2.1 Formal preliminaries

We suppose that the beliefs of an agent are given in the form of propositional sentences. Let At be a propositional signature, *i. e.,* a set of propositional atoms. Let $\mathcal{L}(\mathsf{At})$ be the corresponding propositional language generated by the atoms in At and the connectives $\wedge$ (*and*), $\vee$ (*or*), $\Rightarrow$ (*implication*), and $\neg$ (*negation*). As a notational convenience we assume some arbitrary total order $\ll$ on the elements of $\mathcal{L}(\mathsf{At})$ which is used to enumerate elements of each finite $\Phi \subseteq \mathcal{L}(\mathsf{At})$ in a unique way, cf. [4]. For a finite subset $\Phi \subseteq \mathcal{L}(\mathsf{At})$ the *canonical enumeration* of Φ is the vector $\langle \phi_1, \ldots, \phi_n \rangle$ such that $\{\phi_1, \ldots, \phi_n\} = \Phi$ and $\phi_i \ll \phi_j$ for every $i < j$ with $i, j = 1, \ldots, n$. As $\ll$ is total the canonical enumeration of every finite subset $\Phi \subseteq \mathcal{L}(\mathsf{At})$ is uniquely defined.

We use the operator $\vdash$ to denote classical entailment, *i. e.,* for sets of propositional sentences $\Phi_1, \Phi_2 \subseteq \mathcal{L}(\mathsf{At})$ we say that Φ_2 *follows* from Φ_1, denoted by $\Phi_1 \vdash \Phi_2$, if and only if every $\phi \in \Phi_2$ is entailed by Φ_1 in the classical logical sense. For sentences $\phi, \phi' \in \mathcal{L}(\mathsf{At})$ we write $\phi \vdash \phi'$ instead of $\{\phi\} \vdash \{\phi'\}$. We define the deductive closure $Cn(\cdot)$ of a set of sentences Φ as $Cn(\Phi) = \{\phi \in \mathcal{L}(\mathsf{At}) \mid \Phi \vdash \phi\}$. Two sets of sentences $\Phi, \Phi' \subseteq \mathcal{L}(\mathsf{At})$ are *equivalent*, denoted by $\Phi \equiv \Phi'$, if and only if it holds that $\Phi \vdash \Phi'$ and $\Phi' \vdash \Phi$. We also use the equivalence relation $\cong$ which is defined as $\Phi \cong \Phi'$ if and only if there is a bijection $\sigma : \Phi \to \Phi'$ such that for every $\phi \in \Phi$ it holds that $\phi \equiv \sigma(\phi)$. This means that $\Phi \cong \Phi'$ if Φ and Φ' are *element-wise* equivalent. Note that $\Phi \cong \Phi'$ implies $\Phi \equiv \Phi'$ but not vice versa. In particular, it holds that e. g $\{a \wedge b\} \equiv \{a, b\}$ but $\{a \wedge b\} \not\cong \{a, b\}$. For sentences $\phi, \phi' \in \mathcal{L}(\mathsf{At})$ we write $\phi \sim \phi'$ instead of $\{\phi\} \equiv \{\phi'\}$ if $\sim$ is one of $\{\equiv, \cong\}$. If $\Phi \vdash \perp$ we say that Φ is *inconsistent*.

For a set S let $\mathfrak{P}(S)$ denote the power set of S, *i. e.,* the set of all subsets of S. For a set S let $\mathfrak{P}\mathfrak{P}(S)$ denote the set of multi-sets of S, *i. e.,* the set of all subsets of S where an element may occur more than once. To distinguish sets from multi-sets we use brackets "$\langle$" and "$\rangle$" for the latter.

2.2 Selective Revision

The field of belief revision is concerned with the change of beliefs when more recent or more reliable information is at hand. The most important description of properties of *prioritized* belief change operators are given by Gärdenfors [18, 19], and then by Alchourrón, Gärdenfors and Makinson in their seminal paper [1]. There, the authors consider a model of change in which the epistemic state is represented by a *belief set*, that is, a set of sentences closed under logical consequence, and the epistemic input (the new information) is represented by a single sentence. A *belief set* $\mathbf{K}$ is a subset of $\mathcal{L}(\mathsf{At})$ that is deductively closed, *i. e.,* $\mathbf{K} = Cn(\mathbf{K})$.

Selective Revision [14] is a kind of revision categorized as *non-prioritized*: that is, a revision operator in which the new information is not always accepted. There, the problem of revising a belief set $\mathbf{K}$ with a single sentence α is realized by applying a *transformation function* f to α, obtaining a new sentence α', and then revising $\mathbf{K}$ by α' in a prioritized way. The transformation function f is supposed to determine whether α should be accepted as a whole or whether it should be somewhat weakened. So, a main feature of a selective revision operator is that the new information can be partially accepted.

The selective revision operator on belief sets takes a belief set $\mathbf{K}$ and a sentence α, and produces a new belief set in which 'a selective part' of α can be accepted. More formally, a selective revision operator $\circ$ is defined by the equality $\mathbf{K} \circ \alpha = \mathbf{K} * f(\alpha)$, where $*$ is an AGM revision operator [1] and f is a function, typically with the property $\vdash \alpha \to f(\alpha)$. Fermé and Hansson provide a set of postulates and different constructions for the operator $\circ$, some properties for the function f, and they present representation theorems for three different kinds of selective revision operators. The representation theorems indicate that these constructions provide a fairly faithful extension of the AGM framework to allow for less than total acceptance of new information [14].

Selective revision on belief bases is a generalization of selective revision proposed in [14] and base revision proposed, among others, in [7, 15, 24, 22, 27]. Let $\mathcal{K} \subseteq \mathcal{L}(\mathsf{At})$ be a belief base, and α be a sentence of the language. Then, a selective revision $\circ$ of the belief base $\mathcal{K}$ with respect to α, noted by $\mathcal{K} \circ \alpha$, is a new belief base such that the *success postulate* ($\mathcal{K} \circ \alpha \vdash \alpha$), in general, does not hold.

2.3 Selective Multiple Revision on Belief Bases

In this chapter, we consider the problem of *multiple belief base revision* [29], i.e., revising a finite set of sentences by another such set; cf. also the notions of *multiple change* [16, 26] and *parallel belief revision* [8, 28]. Let $\mathcal{K} \subseteq \mathcal{L}(\mathsf{At})$ be a belief base, $\Phi \subseteq \mathcal{L}(\mathsf{At})$ be some set of sentences, and consider the problem of changing $\mathcal{K}$ in order to entail Φ. If $\mathcal{K} \cup \Phi$ is consistent then there is no need for contracting the existing beliefs and the problem can be solved via *expansion* $\mathcal{K} + \Phi$ which is characterized via $\mathcal{K} \cup \Phi$. If $\mathcal{K} \cup \Phi$ is inconsistent, conflicts arising from the addition of Φ to $\mathcal{K}$ have to be resolved. In general, this means that some of the current beliefs have to be given up in order to come up with a consistent belief base. The AGM framework [1] proposes several basic postulates a revision operator should obey. As we consider belief bases for knowledge representation we start with the corresponding postulates for belief base revision [26] adapted to revision by sets of sentences [11]. Let $*$ be a multiple base revision operator—i. e., if $\mathcal{K}$ and Φ are sets of sentences so is $\mathcal{K} * \Phi$—and consider the following postulates:

Success. $\mathcal{K} * \Phi \vdash \Phi$.

Inclusion. $\mathcal{K} * \Phi \subseteq \mathcal{K} + \Phi$.

Vacuity. If $\mathcal{K} \cup \Phi \nvdash \perp$ then $\mathcal{K} + \Phi \subseteq \mathcal{K} * \Phi$.

Consistency. If Φ is consistent then $\mathcal{K} * \Phi$ is consistent.

Core Retainment. If $\alpha \in (\mathcal{K} \cup \Phi) \setminus (K * \Phi)$ then there is a set H such that $H \subseteq K \cup \Phi$ and H is consistent but $H \cup \{\alpha\}$ is inconsistent.

Relevance. If $\alpha \in (\mathcal{K} \cup \Phi) \setminus (K * \Phi)$ then there is a set H such that $\mathcal{K} * \Phi \subseteq H \subseteq K \cup \Phi$ and H is consistent but $H \cup \{\alpha\}$ is inconsistent.

Success states that the new beliefs in Φ have primacy over beliefs in $\mathcal{K}$. Inclusion determines that the belief base revised by Φ is included in the belief base expanded by Φ. Vacuity establishes that if Φ is consistent with the original belief base, then nothing is removed in the revised belief base. Consistency determines that if Φ is consistent then so is the revised belief base. Core Retainment and Relevance establish the intuition that nothing is removed from the original belief base unless its removal in some way contributes to making the new belief base consistent. It is clear that relevance implies core retainment; the difference among these postulates arises from the construction of contraction operators. If contraction is defined by a kernel contraction operator [23] then core retainment is satisfied; if contraction is defined by a partial meet contraction operator [1] then relevance is satisfied. Since we are presenting a revision operator, it is important to remark that revision operators can be defined from the Levi Identity [30, 20].

Another important property for the framework of [1] is extensionality which can be phrased for multiple base revision as follows:

Extensionality. If $\Phi \equiv \Psi$, then $\mathcal{K} * \Phi \equiv \mathcal{K} * \Psi$.

The above property is not usually considered for the problem of base revision as base revision is motivated by observing explicitly given beliefs and not (only) semantic contents. In particular, for the problem of multiple base revision, satisfaction of *extensionality* imposes that $\mathcal{K} * \{a, b\} \equiv \mathcal{K} * \{a \wedge b\}$ as $\{a, b\} \equiv \{a \wedge b\}$. Identifying the "comma"-operator with the logical "AND"-operator is not always a reasonable thing to do, see e. g. [8, 28] for a discussion. However, we consider the following weakened form of *extensionality*.

Weak Extensionality. If $\Phi \cong \Phi'$ then $\mathcal{K} * \Phi \equiv \mathcal{K} * \Phi'$.

The *weak extensionality* property only states that the outcomes of the revisions $\mathcal{K} * \Phi$ and $\mathcal{K} * \Phi'$ are equivalent if Φ and Φ' are element-wise equivalent.

DEFINITION 3 *A revision operator $*$ is called a* prioritized multiple base revision operator *if $*$ satisfies* success, inclusion, vacuity, consistency, relevance*, and* weak extensionality.

For non-prioritized multiple base revision the properties *inclusion*, *vacuity*, *consistency*, *relevance*, and *weak extensionality* can also be regarded as desirable. This is not the case for *success* in general but we can replace *success* by weakened versions, cf. [25]. We denote with $\circ$ a non-prioritized belief revision operator, i. e., $\mathcal{K} \circ \Phi$ is the non-prioritized revision of $\mathcal{K}$ by Φ. Then consider the following properties for $\circ$, cf. [14].

Weak Success. If $\mathcal{K} \cup \Phi \not\vdash \perp$ then $\mathcal{K} \circ \Phi \vdash \Phi$.

Consistent Expansion. If $\mathcal{K} \not\subseteq \mathcal{K} \circ \Phi$ then $\mathcal{K} \cup (\mathcal{K} \circ \Phi) \vdash \perp$.

Note that *weak success* follows from *vacuity*, and *consistent expansion* follows from *vacuity* and *success*, cf. [14].

DEFINITION 4 *A revision operator* $\circ$ *is called* non-prioritized multiple base revision operator *if* $\circ$ *satisfies* inclusion*,* consistency*,* weak extensionality*,* weak success*, and* consistent expansion*.*

We do not require *relevance* to be satisfied by non-prioritized multiple base revisions as it is hardly achievable in the context of selective revision, see below. For the following, bear in mind that the main difference between a prioritized multiple base revision operator $*$ and a non-prioritized multiple base revision operator $\circ$ is that $\mathcal{K} * \Phi \vdash \Phi$ is required but $\mathcal{K} \circ \Phi \vdash \Phi$ is not.

We adopt the notions of [14] for the problem of *selective multiple belief base revision* and still consider the problem of revising a belief base $\mathcal{K}$ by some set Φ of sentences. Following the ideas of [14] we define the selective multiple base revision.

DEFINITION 5 *Let* $\mathcal{K}$ *be a belief base,* $f_{\mathcal{K}} : \mathfrak{P}(\mathcal{L}(\mathsf{At})) \to \mathfrak{P}(\mathcal{L}(\mathsf{At}))$ *be a transformation function,* $*$ *be some prioritized multiple base revision, and* Φ *be a set of beliefs. Then, the* selective multiple base revision *of* $\mathcal{K}$ *by* Φ*, noted by* $\mathcal{K} \circ \Phi$*, is defined as follows:*

$$\mathcal{K} \circ \Phi = \mathcal{K} * f_{\mathcal{K}}(\Phi)$$

In [14] several properties for transformation functions in the context of belief set revision are discussed which often corresponds to properties of revision operators. We rephrase some of them here slightly to fit the framework of multiple base revision. Let $\mathcal{K} \subseteq \mathcal{L}(\mathsf{At})$ be consistent and let $\Phi, \Phi' \subseteq \mathcal{L}(\mathsf{At})$.

Inclusion. $f_{\mathcal{K}}(\Phi) \subseteq \Phi$.

Weak Inclusion. If $\mathcal{K} \cup \Phi$ is consistent then $f_{\mathcal{K}}(\Phi) \subseteq \Phi$.

Extensionality. If $\Phi \equiv \Phi'$ then $f_{\mathcal{K}}(\Phi) \equiv f_{\mathcal{K}}(\Phi')$.

Consistency Preservation. If Φ is consistent then $f_{\mathcal{K}}(\Phi)$ is consistent.

Consistency. $f_{\mathcal{K}}(\Phi)$ is consistent.

Maximality. $f_{\mathcal{K}}(\Phi) = \Phi$.

Weak Maximality. If $\mathcal{K} \cup \Phi$ is consistent then $f_{\mathcal{K}}(\Phi) = \Phi$.

We also consider the following novel property which corresponds directly to *Weak Extensionality* for multiple base revision operators introduced above.

Weak Extensionality. If $\Phi \cong \Phi'$ then $f_{\mathcal{K}}(\Phi) \cong f_{\mathcal{K}}(\Phi')$.

Not all of the above properties may be desirable for a transformation function that is to be used for selective revision. For example, the property *maximality* states that $f_{\mathcal{K}}$ should not modify the set Φ. Satisfaction of this property makes Definition 5 equivalent to $\mathcal{K} * \Phi$. As $*$ is meant to be a prioritized revision function we lose the possibility for non-prioritized revision.

Note that for *weak extensionality* we demand $f_{\mathcal{K}}(\Phi)$ and $f_{\mathcal{K}}(\Phi')$ to be element-wise equivalent instead of just equivalent (in contrast to the property *weak extensionality* for revision). We do this because $f_{\mathcal{K}}$ is supposed to be applied in the context of base revision which is sensitive to syntactic variants. We introduce the postulate *weak extensionality* for transformation functions with the same motivation as we do for multiple base revision. However, for the case of transformation functions the problem with satisfaction of *extensionality* is more apparent. Consider again $\Phi = \{a, b\}$ and $\Phi' = \{a \wedge b\}$. It follows that $\Phi \equiv \Phi'$ and if $f_{\mathcal{K}}$ satisfies *extensionality* this results in $f_{\mathcal{K}}(\{a, b\}) \equiv f_{\mathcal{K}}(\{a \wedge b\})$. If $f_{\mathcal{K}}$ also satisfies *inclusion* it follows that $f_{\mathcal{K}}(\{a \wedge b\}) \in \{\emptyset, \{a \wedge b\}\}$ and therefore $f_{\mathcal{K}}(\{a, b\}) \in \{\emptyset, \{a, b\}\}$. In general, if $f_{\mathcal{K}}$ satisfies both *inclusion* and *extensionality* it follows that either $f_{\mathcal{K}}(\Phi) = \emptyset$ or $f_{\mathcal{K}}(\Phi) = \Phi$ for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$ (as Φ is equivalent to a Φ' that consists of a single formula that is the conjunction of the formulas in Φ and $f_{\mathcal{K}}(\Phi') = \emptyset$ or $f_{\mathcal{K}}(\Phi') = \Phi'$ due to *inclusion*). As we are interested in a more graded approach to belief revision we want to be able to accept or reject specific pieces of Φ and not just Φ as a whole. Consequently, we consider *weak extensionality* as a desirable property instead of *extensionality.* Note that *extensionality* implies *weak extensionality* as $\Phi \cong \Phi'$ implies $\Phi \equiv \Phi'$.

In [14] several representation theorems are given that characterize non-prioritized belief revision by selective revision via (see Def. 5) and specific properties of $*$ and $f_{\mathcal{K}}$. In particular, it is shown that a reasonable non-prioritized belief revision operator $\circ$ can be characterized by an AGM revision $*$ and a transformation function $f_{\mathcal{K}}$ that satisfies *extensionality*, *consistency preservation*, and *weak maximality.* Note, however, that [14] deals with the problem of revising a belief set by a single sentence. Nonetheless, we can carry over the results of [14] to the problem of multiple base revision and obtain the following result; the proof can be found in [29]).

PROPOSITION 6 *Let* $*$ *be a prioritized multiple base revision operator and let* $f_{\mathcal{K}}$ *satisfy* inclusion*,* weak extensionality*,* consistency preservation*, and* weak maximality*. Then* $\circ$ *defined according to Def. (5) is a non-prioritized multiple base revision operator.*

Note that *relevance* does not hold for $\mathcal{K} \circ \Phi$ defined via Definition 5 in general. Consider for example the transformation function $f^0_{\mathcal{K}}$ defined via $f^0_{\mathcal{K}}(\Phi) = \Phi$ if $\mathcal{K} \cup \Phi$ is consistent and $f^0_{\mathcal{K}}(\Phi) = \emptyset$ otherwise. Then $f^0_{\mathcal{K}}$ satisfies all properties for transformation functions except *maximality*. But it is easy to see that $\mathcal{K} \circ \Phi$ defined via Definition 5 using $f^0_{\mathcal{K}}$ and a prioritized multiple base revision operator $*$ fails to satisfy *relevance*. We leave it to future work to investigate further properties for transformation functions that may enable *relevance* to hold in general.

In the following we aim at implementing a selective multiple base revision using deductive argumentation and go on with introducing the latter.

3 Deductive Argumentation

Argumentation frameworks [2] allow for reasoning with inconsistent information based on the notions of arguments, counterarguments and their relationships. Since the seminal paper [10] interest has grown in research in computational models for argumentation that allow for a coherent procedure for consistent reasoning in the presence of inconsistency. In this work we use the framework of *deductive argumentation* as proposed by Besnard and Hunter [4]. This framework is based on classical propositional logic and is therefore apt for our aim to use argumentation to realize a transformation function f. The central notion of the framework of deductive argumentation is that of an *argument*.

DEFINITION 7 (Argument) *Let* $\Phi \subseteq \mathcal{L}(\mathsf{At})$ *be a set of sentences. An* argument $\mathcal{A}$ *for a sentence* $\alpha \in \mathcal{L}(\mathsf{At})$ *in* Φ *is a tuple* $\mathcal{A} = \langle \Psi, \alpha \rangle$ *with* $\Psi \subseteq \Phi$ *that satisfies*

1. $\Psi \nvdash \perp$*,*
2. $\Psi \vdash \alpha$*, and*
3. *there is no* $\Psi' \subsetneq \Psi$ *with* $\Psi' \vdash \alpha$*.*

For an argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ *we say that* α *is the* claim *of* $\mathcal{A}$ *and* Ψ *is the* support *of* $\mathcal{A}$*.*

Thus, in an argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ for α, the set Ψ is a minimal set entailing α. Given a set $\Phi \subseteq \mathcal{L}(\mathsf{At})$ of sentences there may be multiple arguments for α. As in [4] we are interested in arguments that are most cautious.

DEFINITION 8 (Conservativeness) *An argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ is more* conservative *than an argument $\mathcal{B} = \langle \Phi, \beta \rangle$ if and only if $\Psi \subseteq \Phi$ and $\beta \vdash \alpha$.*

In other words, an $\mathcal{A}$ is more conservative than an argument $\mathcal{B}$ if $\mathcal{A}$ has a smaller support (with respect to set inclusion) and a more general conclusion. An argument $\mathcal{A}$ is *strictly more conservative* than an argument $\mathcal{B}$ if and only if $\mathcal{A}$ is more conservative than $\mathcal{B}$ but $\mathcal{B}$ is not more conservative than $\mathcal{A}$. If $\Phi \subseteq \mathcal{L}(\mathsf{At})$ is inconsistent there are arguments with contradictory claims.

DEFINITION 9 (Undercut) *An argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ is an* undercut *for an argument $\mathcal{B} = \langle \Phi, \beta \rangle$ if and only if $\alpha = \neg(\phi_1 \wedge \ldots \wedge \phi_n)$ for some $\phi_1, \ldots, \phi_n \subseteq \Phi$.*

If $\mathcal{A}$ is an undercut for $\mathcal{B}$ then we also say that $\mathcal{A}$ *attacks* $\mathcal{B}$. In order to consider only those undercuts for an argument that are most general we restrain the notion of undercut as follows.

DEFINITION 10 (Maximally conservative undercut) *An argument $\mathcal{A} = \langle \Psi, \alpha \rangle$ is a* maximally conservative undercut *for an argument $\mathcal{B} = \langle \Phi, \beta \rangle$ if and only if $\mathcal{A}$ is an undercut of $\mathcal{B}$ and there is no undercut $\mathcal{A}'$ for $\mathcal{B}$ that is strictly more conservative than $\mathcal{A}$.*

DEFINITION 11 (Canonical undercut) *An argument $\mathcal{A} = \langle \Psi, \neg(\phi_1 \wedge \ldots \wedge \phi_n) \rangle$ is a* canonical undercut *for an argument $\mathcal{B} = \langle \Phi, \beta \rangle$ if and only if $\mathcal{A}$ is a maximally conservative undercut for $\mathcal{B}$ and $\langle \phi_1, \ldots, \phi_n \rangle$ is the canonical enumeration of Φ.*

It can be shown that it suffices to consider only the canonical undercuts for an argument in order to come up with a reasonable argumentative evaluation of some claim α [4]. Having an undercut $\mathcal{B}$ for an argument $\mathcal{A}$ there may also be an undercut $\mathcal{C}$ for $\mathcal{B}$ which *defends* $\mathcal{A}$. In order to give a proper evaluation of some argument $\mathcal{A}$ we have to consider all undercuts for its undercuts as well, and so on. This leads to the notion of an *argument tree.*

DEFINITION 12 (Argument tree) *Let $\alpha \in \mathcal{L}(\mathsf{At})$ be some sentence and let $\Phi \subseteq \mathcal{L}(\mathsf{At})$ be a set of sentences. An* argument tree *$\tau_\Phi(\alpha)$ for α in Φ is a tree where the nodes are arguments and that satisfies*

1. *the root is an argument for* α *in* Φ*,*

2. *for every path* $[\langle\Phi_1,\alpha_1\rangle,\ldots,\langle\Phi_n,\alpha_n\rangle]$ *in* $\tau_\Phi(\alpha)$ *it holds that* $\Phi_n \not\subseteq \Phi_1 \cup \ldots \cup \Phi_{n-1}$*, and*

3. *the children* $\mathcal{B}_1,\ldots,\mathcal{B}_m$ *of a node* $\mathcal{A}$ *consist of all canonical undercuts for* $\mathcal{A}$ *such that condition (2) above is not violated when these canonical undercuts are added as children.*

Let $\mathcal{T}(\mathsf{At})$ *be the set of all argument trees.*

An argument tree is a concise representation of the relationships between different arguments that favor or reject some argument $\mathcal{A}$. In order to evaluate whether a claim α can be justified we have to consider all argument trees for α and all argument trees for $\neg\alpha$. For an argument tree τ let $\mathsf{root}(\tau)$ denote the root node of τ. Furthermore, for a node $\mathcal{A}\in\tau$ let $\mathsf{ch}_\tau(\mathcal{A})$ denote the children of $\mathcal{A}$ in τ and $\mathsf{ch}_\tau^{\mathcal{T}}(\mathcal{A})$ denote the set of sub-trees rooted at a child of $\mathcal{A}$.

DEFINITION 13 (Argument structure) *Let* $\alpha \in \mathcal{L}(\mathsf{At})$ *be some sentence and let* $\Phi \subseteq \mathcal{L}(\mathsf{At})$ *be a set of sentences. The* argument structure $\Gamma_\Phi(\alpha)$ *for* α *with respect to* Φ *is the tuple* $\Gamma_\Phi(\alpha) = (\mathcal{P},\mathcal{C})$ *such that* $\mathcal{P}$ *is the set of argument trees for* α *in* Φ *and* $\mathcal{C}$ *is the set of arguments trees for* $\neg\alpha$ *in* Φ*.*

The argument structure $\Gamma_\Phi(\alpha)$ of a $\alpha \in \mathcal{L}(\mathsf{At})$ gives a complete picture of the reasons for and against α. The argument structure has to be evaluated in order to determine the status of sentences. We introduce the powerful evaluation mechanisms from [4] and give examples of how adequate and simple instantiations can be realized.

DEFINITION 14 (Categorizer) *A* categorizer γ *is a function* $\gamma : \mathcal{T}(\mathsf{At}) \to \mathbb{R}$.

A categorizer is meant to assign a value to an argument tree τ depending on how strongly this argument tree favors the root argument. In particular, the larger the value of $\gamma(\tau)$ the better justification of believing in the claim of the root argument. For an argument structure $\Gamma_\Phi(\alpha) = (\{\tau_1^p,\ldots,\tau_n^p\},\{\tau_1^c,\ldots,\tau_m^c\})$ and a categorizer γ we abbreviate

$$\gamma(\Gamma_\Phi(\alpha)) = (\langle\gamma(\tau_1^p),\ldots,\gamma(\tau_n^p)\rangle,\langle\gamma(\tau_1^c),\ldots,\gamma(\tau_m^c)\rangle) \in \mathfrak{PP}(\mathbb{R})\times\mathfrak{PP}(\mathbb{R})\,.$$

DEFINITION 15 (Accumulator) *An* accumulator κ *is a function* $\kappa : \mathfrak{PP}(\mathbb{R})\times\mathfrak{PP}(\mathbb{R}) \to \mathbb{R}$.

An accumulator is meant to evaluate the categorization of argument trees for or against some sentence α.

DEFINITION 16 (Acceptance) *We say that a set of sentences* $\Phi \subseteq \mathcal{L}(\mathsf{At})$ accepts *a sentence* α *with respect to a categorizer* γ *and an accumulator* κ*, denoted by*

$$\Phi \mathrel{|\!\sim}_{\kappa,\gamma} \alpha \text{ if and and only if } \kappa(\gamma(\Gamma_\Phi(\alpha))) > 0$$

If Φ *does not accept* α *with respect to* γ *and* κ *(*$\Phi \mathrel{|\!\not\sim}_{\kappa,\gamma} \alpha$*) we say that* Φ rejects α *with respect to* γ *and* κ*.*

Some simple instances of categorizers and accumulators are as follows.

EXAMPLE 17 *Let* τ *be some argument tree. The classical evaluation of an argument tree—as* e. g., *employed in* Defeasible Logic Programming *[17]—is as follows: each leaf of the tree is considered "undefeated"; an inner node is "undefeated" if all its children are "defeated" and "defeated" if there is at least one child that is "undefeated". This evaluation can be formalized by defining the* classical categorizer γ_0 *recursively via*

$$\gamma_0(\tau) = \begin{cases} 1 & \text{if } \mathsf{ch}_\tau(\mathsf{root}(\tau)) = \emptyset \\ 1 - \max\{\gamma_0(\tau') \mid \tau' \in \mathsf{ch}_\tau^{\mathcal{T}}(\mathsf{root}(\tau))\} & \text{otherwise} \end{cases}$$

Furthermore, a simple accumulator κ_0 *can be defined via*

$$\kappa_0(\langle N_1,\ldots,N_n\rangle, \langle M_1,\ldots,M_m\rangle) = N_1 + \ldots + N_n - M_1 - \ldots - M_m\,.$$

For example, a set of sentences $\Phi \subseteq \mathcal{L}(\mathsf{At})$ *accepts a sentence* α *with respect to* γ_0 *and* κ_0 *if and only if there are more argument trees for* α *where the root argument is undefeated than argument trees for* $\neg\alpha$ *where the root argument is undefeated.* ■

More examples of categorizers and accumulators can be found in [4]. Using those notions we are able to state for every sentence $\phi \in \Phi$ whether ϕ is accepted in Φ or not, depending on the arguments that favor α and those that reject α.

4 Selective Revision by Deductive Argumentation

Using the deductive argumentation framework presented in the previous section one is able to decide for each sentence $\alpha \in \Phi$ whether α is justifiable with respect to Φ. Note that the framework of deductive argumentation heavily depends on the actual instances of categorizer and accumulator. In the following we only consider categorizers and accumulators that comply with the following minimal requirements.

DEFINITION 18 (Well-behaving categorizer) *A categorizer* γ *is called* well-behaving *if* $\gamma(\tau) > \gamma(\tau')$ *whenever* τ *consists only of one single node and* τ' *consists of at least two nodes.*

In other words, a categorizer γ is well-behaving if the argument tree that has no undercuts for its root is considered the best justification for the root.

DEFINITION 19 (Well-behaving accumulator) *An accumulator* κ *is called* well-behaving *if and only if* $\kappa((\mathcal{P},\mathcal{C})) > 0$ *whenever* $\mathcal{P} \neq \emptyset$ *and* $\mathcal{C} = \emptyset$*.*

This means that if there are no arguments against a claim α and at least one argument for α in Φ then α should be accepted in Φ. Note that both γ_0 and κ_0 are well-behaving as well as all categorizers and accumulators considered in [4]. Furthermore, if Φ is consistent then every sentence $\alpha \in \Phi$ is accepted by Φ with respect to every well-behaving categorizer and well-behaving accumulator.

Let $\mathcal{K} \subseteq \mathcal{L}(\mathsf{At})$ be a consistent set of sentences, and let γ be some well-behaving categorizer and κ be some well-behaving accumulator. We consider again a selective revision $\circ$ of the form introduced in Definition 5. In order to determine the outcome of the non-prioritized revision $\mathcal{K} \circ \Phi$ for some $\Phi \subseteq \mathcal{L}(\mathsf{At})$ we implement a transformation function f that checks for every sentence $\alpha \in \Phi$ whether α is accepted in $\mathcal{K} \cup \Phi$. Note that although $\mathcal{K}$ is consistent the union $\mathcal{K} \cup \Phi$ is not necessarily consistent which gives rise to an argumentative evaluation. In the following, we consider *two* different transformation functions based on deductive argumentation.

DEFINITION 20 (Skeptical Transformation Function) *We define the* skeptical transformation function $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}$ *via*

$$\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) = \{\alpha \in \Phi \mid \mathcal{K} \cup \Phi \mathrel{|\!\sim}_{\kappa,\gamma} \alpha\}$$

for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$*.*

DEFINITION 21 (Credulous Transformation Function) *We define the* credulous transformation function $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}$ *via*

$$\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}(\Phi) = \{\alpha \in \Phi \mid \mathcal{K} \cup \Phi \mathrel{|\!\not\sim}_{\kappa,\gamma} \neg\alpha\}$$

for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$*.*

In other words, the value of $\mathsf{S}_{\mathcal{K}}^{\gamma,\kappa}(\Phi)$ consists of those sentences of Φ that are accepted in $\mathcal{K} \cup \Phi$ and the value of $\mathsf{C}_{\mathcal{K}}^{\gamma,\kappa}(\Phi)$ consists of those sentences of Φ that are not rejected in $\mathcal{K} \cup \Phi$. There is a subtle difference in the behavior of these two transformation functions as the following example shows.

EXAMPLE 22 *Let $\mathcal{K}_1 = \{a\}$ and $\Phi_1 = \{\neg a\}$. Note that there is exactly one argument tree τ_1 for $\neg a$ and one argument tree τ_2 for a in $\mathcal{K}_1 \cup \Phi_1$. In τ_1 the root is the argument $\mathcal{A} = \langle\{\neg a\}, \neg a\rangle$ which has the single canonical undercut $\mathcal{B} = \langle\{a\}, a\rangle$. In τ_2 the situation is reversed and the root of τ_2 is the argument $\mathcal{B}$ which has the single canonical undercut $\mathcal{A}$. Therefore, the argument structure for $\neg a$ is given via $\Gamma_{\mathcal{K}\cup\Phi}(\neg a) = (\{\tau_1\}, \{\tau_2\})$. This implies that $\gamma_0(\tau_1) = \gamma_0(\tau_2) = 0$ and $\kappa_0(\gamma_0(\Gamma_{\mathcal{K}\cup\Phi}(a))) = \kappa_0(\langle 0, 0\rangle) = 0$. Thus, $\mathcal{K} \cup \Phi$ is undecided about both $\neg a$ and a. Consequently, it follows that*

$$\mathsf{S}^{\gamma_0,\kappa_0}_{\mathcal{K}_1}(\Phi_1) = \emptyset \qquad\qquad \mathsf{C}^{\gamma_0,\kappa_0}_{\mathcal{K}_1}(\Phi_1) = \{\neg a\}\,.$$

■

Let $*$ be some (prioritized) multiple base revision operator, γ some categorizer, and κ some accumulator. Using the skeptical transformation function we can define the *skeptical argumentative revision* $\circ^{\gamma,\kappa}_{S}$ following Definition 5 via

$$\mathcal{K} \circ^{\gamma,\kappa}_{S} \Phi = \mathcal{K} * \mathsf{S}^{\gamma,\kappa}_{\mathcal{K}}(\Phi) \tag{1}$$

for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$ and using the credulous transformation function we can define the *credulous argumentative revision* $\circ^{\gamma,\kappa}_{C}$ via

$$\mathcal{K} \circ^{\gamma,\kappa}_{C} \Phi = \mathcal{K} * \mathsf{C}^{\gamma,\kappa}_{\mathcal{K}}(\Phi) \tag{2}$$

for every $\Phi \subseteq \mathcal{L}(\mathsf{At})$.

EXAMPLE 23 *We continue Example 22. Let $*$ be some prioritized multiple base revision. Then it follows that $\mathcal{K}_1 \circ^{\gamma_0,\kappa_0}_{S} \Phi_1 = \{a\}$ and $\mathcal{K}_1 \circ^{\gamma_0,\kappa_0}_{C} \Phi_1 = \{\neg a\}$.* ■

We now investigate the formal properties of the transformation functions $\mathsf{S}^{\gamma,\kappa}_{\mathcal{K}}$ and $\mathsf{C}^{\gamma,\kappa}_{\mathcal{K}}$ and the resulting revision operators $\circ^{\gamma,\kappa}_{S}$ and $\circ^{\gamma,\kappa}_{C}$.

PROPOSITION 24 *Let γ be a well-behaving categorizer and κ be a well-behaving accumulator. Then the transformation functions $\mathsf{S}^{\gamma,\kappa}_{\mathcal{K}}$ and $\mathsf{C}^{\gamma,\kappa}_{\mathcal{K}}$ satisfy* inclusion, weak inclusion, weak extensionality, consistency preservation *and* weak maximality.

Proof.

Inclusion. *This is satisfied by definition as for $\alpha \in \mathsf{S}^{\gamma,\kappa}_{\mathcal{K}}(\Phi)$ and each $\alpha \in \mathsf{C}^{\gamma,\kappa}_{\mathcal{K}}(\Phi)$ it follows $\alpha \in \Phi$.*

Weak Inclusion. *This follows directly from the satisfaction of* inclusion.

Weak Extensionality. *Let $\Phi \cong \Phi'$ and let $\sigma : \Phi \to \Phi'$ be a bijection such that for every $\phi \in \Phi$ it holds that $\phi \equiv \sigma(\phi)$. We extend σ to $\mathcal{K}$ via $\sigma(\psi) = \psi$ for every $\psi \in \mathcal{K}$. If $\Psi \subseteq \mathcal{K} \cup \Phi$ we abbreviate*

$$\sigma(\Psi) = \bigcup_{\psi \in \Psi} \{\sigma(\psi)\}\,.$$

Let $\langle \Psi, \phi \rangle$ be an argument for some $\phi \in \Phi$ with respect to $\mathcal{K} \cup \Phi$. Then $\langle \sigma(\Psi), \sigma(\phi) \rangle$ is an argument for $\sigma(\phi)$ in $\mathcal{K} \cup \Phi'$. It follows that if τ is an argument tree for $\langle \Psi, \phi \rangle$ in $\mathcal{K} \cup \Phi$ then τ' is an argument tree for $\langle \sigma(\Psi), \sigma(\phi) \rangle$ in $\mathcal{K} \cup \Phi'$ where τ' is obtained from τ by replacing each sentence ϕ with $\sigma(\phi)$. This generalizes also to argument structures and it follows that

$$\kappa(\gamma(\Gamma_{\mathcal{K} \cup \Phi}(\phi))) = \kappa(\gamma(\Gamma_{\mathcal{K} \cup \Phi'}(\sigma(\phi))))\,.$$

Hence, $\phi \in \mathsf{S}^{\gamma;\kappa}_{\mathcal{K}}(\Phi)$ if and only if $\sigma(\phi) \in \mathsf{S}^{\gamma;\kappa}_{\mathcal{K}}(\Phi')$ for every $\phi \in \Phi$. It follows that $\mathsf{S}^{\gamma;\kappa}_{\mathcal{K}}(\Phi) \cong \mathsf{S}^{\gamma;\kappa}_{\mathcal{K}}(\Phi')$. The same is true for $\mathsf{C}^{\gamma;\kappa}_{\mathcal{K}}$.

Consistency Preservation. *Every subset of a consistent set of sentences is consistent and, due to* inclusion*, it holds that $\mathsf{S}^{\gamma;\kappa}_{\mathcal{K}}(\Phi), \mathsf{C}^{\gamma;\kappa}_{\mathcal{K}}(\Phi) \subseteq \Phi$ with consistent Φ.*

Weak Maximality. *If $\mathcal{K} \cup \Phi$ is consistent then for all arguments for a sentence $\alpha \in \Phi$ there do not exist any undercuts as these would have to entail the negation of some sentence of the argument for α which implies inconsistency of $\mathcal{K} \cup \Phi$. The argument structure $\Gamma_\Phi(\alpha) = (\mathcal{P}, \mathcal{C})$ consists of one or more single node trees $\mathcal{P}$ and $\mathcal{C} = \emptyset$. As both γ and κ are well-behaving it follows that $\kappa(\gamma(\Gamma_\Phi(\alpha))) > 0$ for each $\alpha \in \Phi$ and therefore $\mathsf{S}^{\gamma;\kappa}_{\mathcal{K}}(\Phi) = \Phi$ and $\mathsf{C}^{\gamma;\kappa}_{\mathcal{K}}(\Phi) = \Phi$.*

■

In particular, note that both $\mathsf{S}^{\gamma;\kappa}_{\mathcal{K}}$ and $\mathsf{C}^{\gamma;\kappa}_{\mathcal{K}}$ do not satisfy either *consistency* or *maximality* in general.

COROLLARY 25 *Let γ be a well-behaving categorizer and κ be a well-behaving accumulator. Then both $\circ^{\gamma;\kappa}_{S}$ and $\circ^{\gamma;\kappa}_{C}$ are non-prioritized multiple base revision operators.*

Proof. *This follows directly from Propositions 6 and 24.* ■

EXAMPLE 26 *We continue Example 1 which considered* $\mathsf{At} = \{s, h, l, m, f, v\}$ *with the following informal interpretations.*

s : *Anna is a surf fanatic*
h : *Anna travels to Hawaii*
f : *Anna has financial problems*
l : *Anna takes a loan*
m : *Anna has a lot of money*
v : *There is volcano activity on Hawaii*

Now consider Anna's belief base $\mathcal{K}_1$ *given via*

$$\mathcal{K}_1 = \{s, \quad s \Rightarrow h, \quad l, \quad l \Rightarrow m, \quad m \Rightarrow h, \quad m \Rightarrow \neg f\}\,.$$

Note that $\mathcal{K}_1 \vdash h$*, i. e. Anna intends to go to Hawaii. As it was mentioned in Example 1, consider the new information* $\Phi_1 = \{f, \quad f \Rightarrow \neg h, \quad v, \quad v \Rightarrow \neg h\}$ *stemming from communication with Anna's mother. In* Φ_1 *the mother of Anna tells her not to travel to Hawaii.*

As one can see there a several arguments for and against h *in* $\mathcal{K}_1 \cup \Phi_1$*, e. g.,* $\langle s, s \Rightarrow h, h\rangle$*,* $\langle f, f \Rightarrow \neg h, \neg h\rangle$*. We do not go into details regarding the argumentative evaluation of the sentences in* Φ_1 *(Example 34 gives a complete description where these kind of details are shown). We only note that* $\mathcal{K}_1 \cup \Phi_1$ *accepts* $f \Rightarrow \neg h$*, but rejects* f*,* v*, and* $v \Rightarrow \neg h$ *with respect to* γ_0 *and* κ_0*. Furthermore,* $\mathcal{K}_1 \cup \Phi_1$ *accepts* $\neg f$ *and rejects* $\neg v$ *and* $\neg(v \Rightarrow \neg h)$ *with respect to* γ_0 *and* κ_0 *which means that both* v *and* $v \Rightarrow \neg h$ *are credulously accepted. Consequently, the values of* $\mathsf{S}^{\gamma_0,\kappa_0}_{\mathcal{K}_1}(\Phi_1)$ *and* $\mathsf{C}^{\gamma_0,\kappa_0}_{\mathcal{K}_1}(\Phi_1)$ *are given via*

$$\mathsf{S}^{\gamma_0,\kappa_0}_{\mathcal{K}_1}(\Phi_1) = \Phi_1 \setminus \{f, v, v \Rightarrow \neg h\} \quad \text{and} \quad \mathsf{C}^{\gamma_0,\kappa_0}_{\mathcal{K}_1}(\Phi_1) = \Phi_1 \setminus \{f\}\,.$$

Let $*$ *be some prioritized multiple base revision operator and define* $\circ^{\gamma_0,\kappa_0}_S$ *and* $\circ^{\gamma_0,\kappa_0}_C$ *via (1) and (2), respectively. Then some possible revisions of* $\mathcal{K}_1$ *with* Φ_1 *are given via*

$$\mathcal{K}_1 \circ^{\gamma_0,\kappa_0}_S \Phi_1 = \{s, \quad s \Rightarrow h, \quad l, \quad l \Rightarrow m, \quad m \Rightarrow h, \quad m \Rightarrow \neg f, \; f \Rightarrow \neg h\}$$
$$\mathcal{K}_1 \circ^{\gamma_0,\kappa_0}_C \Phi_1 = \{s, \; l \Rightarrow m, \; m \Rightarrow h, \; m \Rightarrow \neg f, \; f \Rightarrow \neg h, \; v \Rightarrow \neg h, \; v\}\,.$$

Note that it holds that $\mathcal{K}_1 \circ^{\gamma_0,\kappa_0}_S \Phi_1 \vdash h$ *and* $\mathcal{K}_1 \circ^{\gamma_0,\kappa_0}_C \Phi_1 \vdash \neg h$*.* ■

For the evaluation of our approach the sophisticated algorithms presented in [3] can be used. However, the underlying problems of deciding whether a set of propositional formulae is consistent is NP-complete and deciding whether it entails a given formula is co-NP-complete [21] such that no generally efficient implementation can be expected.

5 Argumentative Credibility-based Revision

In the previous sections we have shown how argumentation and selective revision can be combined to obtain a non-prioritized revision operator where the incoming information can be evaluated in order to decide if it is accepted or rejected. As shown above, the proposed operator only accepts new information if the information is justifiable with respect to an argumentative evaluation.

Below we will integrate the approach introduced in Section 4 in a multi-agent scenario with information stemming from different agents with different degrees of credibility. We will extend the approach of selective revision by deductive argumentation for this setting by including credibility information in the argumentative process. New information is evaluated based on the credibility of the source in combination with all arguments favoring and opposing the new information. The evaluation process determines which part of the new information is to be accepted for revision and thereupon incorporated into the belief base by an appropriate revision operator.

5.1 Credibility-based Epistemic Models

We continue with developing an epistemic model for an agent in a multi-agent environment that takes the credibilities of other agents into account. Our formalization is based on [33]. Let $\mathbb{A} = \{A_1, \ldots, A_n\}$ be a finite set of agents.

DEFINITION 27 *If* $\phi \in \mathcal{L}(\mathsf{At})$ *and* $A \in \mathbb{A}$ *then* $A{:}\phi$ *is called an* information object. *Let* $\mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ *denote the set of all information objects wrt.* $\mathcal{L}(\mathsf{At})$ *and* $\mathbb{A}$.

An information object $A:\phi$ states that ϕ has been uttered by A. For $\mathcal{I} \subseteq \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ we abbreviate $\mathsf{Form}(\mathcal{I}) = \{\phi \mid A{:}\phi \in \mathcal{I}\}$. We extend the operator $\mathsf{Cn}()$ to $\mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ by defining $\mathsf{Cn}(\mathcal{I}) = \mathsf{Cn}(\mathsf{Form}(\mathcal{I}))$. Note that we do not consider nested information objects such as "A said that A' said that ϕ" to keep things simple. We leave this issue for future work.

REMARK 28 *Although the framework of deductive argumentation from the previous section has been phrased for the language* $\mathcal{L}(\mathsf{At})$ *we adopt the notions in the same manner for* $\mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ *by ignoring the annotated sources. For example, if* $\mathcal{I} \subseteq \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ *and* $A{:}\phi \in \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ *then we say that* $\langle \mathcal{I}, \phi \rangle$ *is an argument whenever* $\langle \mathsf{Form}(\mathcal{I}), \phi \rangle$ *is an argument.*

For $\mathcal{I} \subseteq \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ with $\mathsf{Form}(\mathcal{I}) \nvdash \perp$ and a total preorder $\leq$ on $\mathbb{A}$ (called *credibility order*) the tuple $(\mathcal{I}, \leq)$ is called a *belief base*. If $\mathcal{K}_A = (\mathcal{I}_A, \leq_A)$ is the belief base of an agent A then $A' \leq_A A''$ means that A believes that

A'' is at least as credible as A'. The strict relation $<_A$ and the equivalence relation $\equiv_A$ are defined as usual.

EXAMPLE 29 *Let $\mathbb{A} = \{A_1, A_2, A_3\}$ be a set of agents and consider the belief base $\mathcal{K}_{A_1} = (\mathcal{I}_{A_1}, \leq_{A_1})$ of agent A_1 given via*

$$\mathcal{I}_{A_1} = \{A_1{:}\neg b, A_2{:}a, A_3{:}a \Rightarrow \neg b, A_3{:}c\}$$
$$\leq_{A_1} = A_1 <_{A_1} A_2 <_{A_1} A_3 \quad .$$

Observe that according to $\mathcal{K}_{A_1}$, A_1 believes that c has been uttered by A_3. Furthermore, A_1 believes that A_2 is less credible than A_3 and that himself is less credible than A_2.

Let $A \in \mathbb{A}$ be an agent and let $\mathcal{K}_A = (\mathcal{I}_A, \leq_A)$ be its belief base. The credibility order $\leq_A$ can be used to specify a preference relation among arguments. Let $\langle \mathcal{I}_1, \phi_1 \rangle, \langle \mathcal{I}_2, \phi_2 \rangle$ be two arguments with $\mathcal{I}_1, \mathcal{I}_2 \subseteq \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$. Then $\langle \mathcal{I}_1, \phi_1 \rangle$ is *as least as preferred* as $\langle \mathcal{I}_2, \phi_2 \rangle$ by A, denoted by $\langle \mathcal{I}_2, \phi_2 \rangle \preceq_A \langle \mathcal{I}_1, \phi_1 \rangle$ if and only if for all $B : \phi \in \mathcal{I}_1$ there is a $B' : \phi' \in \mathcal{I}_2$ such that $B' \leq_A B$. In other words, it holds $\langle \mathcal{I}_2, \phi_2 \rangle \preceq_A \langle \mathcal{I}_1, \phi_1 \rangle$ if and only if the least credible source in $\mathcal{I}_1$ is at least as credible as the least credible source of $\mathcal{I}_2$.

EXAMPLE 30 *Consider $\mathcal{K}_{A_1}$ of Example 29. Let $\langle \mathcal{I}_1, \neg b \rangle$ and $\langle \mathcal{I}_2, c \rangle$ be two arguments with $\mathcal{I}_1 = \{A_2{:}a, A_3{:}a \Rightarrow \neg b\}$ and $\mathcal{I}_2 = \{A_3{:}c\}$. According to $<_{A_1}$, A_2 is less credible than A_3 ($A_2 <_{A_1} A_3$) hence $\langle \mathcal{I}_1, \neg b \rangle \prec_{A_1} \langle \mathcal{I}_2, c \rangle$.*

5.2 Credibility-based Revision operation

Consider a multi-agent system with agents $\mathbb{A} = \{A_1, \ldots, A_n\}$ where each agent A_i ($i = 1, \ldots, n$) maintains its own belief base $\mathcal{K}_{A_i} = (\mathcal{I}_{A_i}, \leq_{A_i})$. That is, each agent has some subjective beliefs consisting of individual pieces of information annotated with the source of this information (possibly the agent itself) and some subjective ordering on the credibility of the agents in the system (including itself). When an agent A_j sends some pieces of information $\mathcal{I} \subseteq \mathcal{I}_{A_j}$ to some agent A_i the agent A_i has to deliberate on how to react to receiving $\mathcal{I}$. Clearly, A_i should not blindly—*i. e.,* in a prioritized fashion—revise $\mathcal{I}_{A_i}$ by $\mathcal{I}$ but take into account the credibility of A_j wrt. $\leq_{A_i}$. Furthermore, as $\mathcal{I}$ may contain an information object $A_k : \phi$ with $A_k \neq A_j$, *i. e.,* agent A_j forwards some information from A_k to A_i, agent A_i should also consider the credibility of A_k.

Our approach follows the ideas of Section 4 but also incorporates the role of credibilities. On receiving some pieces of information $\mathcal{I} \subseteq \mathcal{I}_{A_j}$ from some agent A_j agent A_i evaluates each $A : \phi \in \mathcal{I}$ by an argumentation procedure that results in either accepting or rejecting $A:\phi$ for revision. This argumentation procedure is regulated by agent A_i's assessment of the credibilities of the sources of information. In particular, information that comes from a more credible source is preferred to information that comes from a less credible source. For this, we extend our definition of the classical categorizer from Example 17 as follows.

DEFINITION 31 *Let* $\mathcal{K}_A = (\mathcal{I}_A, \leq_A)$ *be the belief base of an agent* A*, let* $\mathcal{I} \subseteq \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$*, and let* τ *be some argument tree for* $A'{:}\phi$ *in* $\mathcal{I}$*. Then define the* credibility categorizer γ^c_A *for* A *through* $\gamma^c_A(\tau) = 1$ *if* $\mathsf{ch}_\tau(\mathsf{root}(\tau)) = \emptyset$ *and through*

$$\gamma^c_A(\tau) = 1 - \max\{\gamma^c_A(\tau') \mid \tau' \in \mathsf{ch}^{\mathcal{T}}_\tau(\mathsf{root}(\tau)) \text{ and } \mathsf{root}(\tau) \preceq_A \mathsf{root}(\tau')\}$$

otherwise.

Note that the credibility categorizer extends the classical categorizer as defined in Example 17 as he takes the subjective credibility order of agent A into account by only considering those sub-trees of a node where the root argument is at least as preferred as the node itself.

EXAMPLE 32 *Let* $\mathbb{A} = \{A_1, A_2, A_3\}$ *be a set of agents and consider the belief base* $\mathcal{K}_{A_1} = (\mathcal{I}_{A_1}, \leq_{A_1})$ *of agent* A_1 *where* $\mathcal{I}_{A_1} = \{A_2{:}b, A_3{:}c\}$ *and* $<_{A_1} = A_1 <_{A_1} A_2 <_{A_1} A_3$*. Let* $\mathcal{I} = \{A_3{:}a \Rightarrow \neg b, A_2{:}a\}$*. Note that there is exactly one argument tree* τ_1 *for* $a \Rightarrow \neg b$ *and one argument tree* τ_2 *for* $a \wedge b$ *in* $\mathcal{I}_{A_1} \cup \mathcal{I}$*. In* τ_1 *the root is the argument* $\mathcal{A} = \langle\{A_3{:}a \Rightarrow \neg b\}, a \Rightarrow \neg b\rangle$ *which has the single canonical undercut* $\mathcal{B} = \langle\{A_2{:}a, A_2{:}b\}, a \wedge b\rangle$*. In* τ_2 *the situation is reversed and the root of* τ_2 *is the argument* $\mathcal{B}$ *which has the single canonical undercut* $\mathcal{A}$*. Therefore, the argument structure for* $a \Rightarrow \neg b$ *is given via* $\Gamma_{\mathcal{I}_{A_1} \cup \mathcal{I}}(a \Rightarrow \neg b) = (\{\tau_1\}, \{\tau_2\})$*. We can see these argument trees in Figure 1. In* τ_1 *one can see that the only child of* $\mathcal{A}$ *is not considered when evaluating with* $\gamma^c_{A_1}$ *because* A_2 *is less credible than* A_3 *according to* A_1*. For this reason* $\gamma^c_{A_1}(\tau_1) = 1$*. However, in* τ_2 *the situation is reversed and* $\mathcal{B}$ *is considered by* $\gamma^c_{A_1}$*. For this reason* $\gamma^c_{A_1}(\tau_2) = 0$*.*

We use the credibility categorizer to evaluate new information $\mathcal{I} \subseteq \mathcal{I}_{A_j}$ received by an agent A_i from agent A_j on an argumentative basis and by taking credibilities into account. As before, we say that an agent A_i with belief base $\mathcal{K}_{A_i} = (\mathcal{I}_{A_i}, \leq_{A_i})$ *accepts* an information object $A{:}\phi \in \mathcal{I}$ wrt. $\mathcal{I}$ if and only if

$$\kappa^c(\mathcal{P}, \mathcal{C}) = \sum_{\tau \in \mathcal{P}} \gamma^c_{A_i}(\tau) - \sum_{\tau \in \mathcal{C}} \gamma^c_{A_i}(\tau) > 0 \tag{3}$$

$$\langle\{A_3{:}a \Rightarrow \neg b\}, a \Rightarrow \neg b\rangle \qquad \langle\{A_2{:}a, A_2{:}b\}, a \wedge b\rangle$$

$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

$$\langle\{A_2{:}a, A_2{:}b\}, a \wedge b\rangle \qquad \langle\{A_3{:}a \Rightarrow \neg b\}, a \Rightarrow \neg b\rangle$$

Figure 1. Argument trees in Example 32

where $\Gamma_{\mathcal{I}_{A_i}\cup\mathcal{I}}(A{:}\phi) = (\mathcal{P}, \mathcal{C})$ is the argument structure for $A{:}\phi$ wrt. $\mathcal{I}_{A_i} \cup \mathcal{I}$, cf. the definition of the simple accumulator in Example 17. Equation (3) means that A_i accepts $A:\phi$ if there are more reasons to believe in ϕ as there are to believe in $\neg\phi$. Using the notion of acceptance we define transformation functions C_{A_i} and S_{A_i} for agent A_i via

$$C_{A_i}(\mathcal{I}) = \{A{:}\phi \in \mathcal{I} \mid A_i \text{ accepts } A{:}\phi \text{ wrt. } \mathcal{I}\}$$
$$S_{A_i}(\mathcal{I}) = \{A{:}\phi \in \mathcal{I} \mid A_i \text{ does not accept } A'{:}\neg\phi \text{ wrt. } \mathcal{I} \text{ for some } A'\}$$

Note that—in contrast to the transformation functions discussed before—the codomains of C_{A_i} and S_{A_i} are subsets of $\mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ instead of $\mathcal{L}(\mathsf{At})$.

We now turn to the issue of revising $\mathcal{I}_{A_i}$ in a prioritized fashion by $C_{A_i}(\mathcal{I})$ and $S_{A_i}(\mathcal{I})$, respectively. We do this by exploiting the Levi-identity for belief revision [1], *i. e.,* by first contracting $\mathcal{I}_{A_i}$ by the complement of $C_{A_i}(\mathcal{I})$ ($S_{A_i}(\mathcal{I})$), which is to be defined, and then expanding by $C_{A_i}(\mathcal{I})$ ($S_{A_i}(\mathcal{I})$). Let $-$ be some belief base contraction—*e. g.,* a kernel contraction [26]—and define a contraction $-_b$ on $\mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ for finite $\mathcal{I} \in \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ and $\phi \in \mathcal{L}(\mathsf{At})$ through

$$\mathcal{I} -_b \phi = \{A{:}\phi' \in \mathcal{I} \mid \phi' \in \mathsf{Form}(\mathcal{I}) - \phi\} \quad .$$

Then, for finite $\mathcal{I}, \mathcal{I}' \in \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$ with $\mathsf{Form}(\mathcal{I}) \nvdash \perp$ define a (prioritized) revision $*$ through

$$\mathcal{I} * \mathcal{I}' = (\mathcal{I} -_b \bigvee_{\phi \in \mathsf{Form}(\mathcal{I}')} \neg\phi) \cup \mathcal{I}' \tag{4}$$

and (non-prioritized) revisions $\circ_A^C$ and $\circ_A^S$ wrt. an agent A through

$$\mathcal{I} \circ_A^C \mathcal{I}' = \mathcal{I} * C_A(\mathcal{I}')$$
$$\mathcal{I} \circ_A^S \mathcal{I}' = \mathcal{I} * S_A(\mathcal{I}')$$

As we stated above, to revise $\mathcal{I}_{A_i}$ for a set of information objects $\mathcal{I}$, we should contract $\mathcal{I}_{A_i}$ by the complement of $\mathcal{I}$. For a given set of information objects $\mathcal{I}$ where $\mathsf{Form}(\mathcal{I}) = \{\phi_1, \ldots, \phi_n\}$, the complement of $\mathcal{I}$ is

$\bigvee_{\phi \in \mathsf{Form}(\mathcal{I})} \neg\phi$. However, defining the complement of $\mathcal{I}$ as $\{\neg\phi_1, \ldots, \neg\phi_n\}$ and using a multiple contraction operator as in [13] would not be sufficient as the following example illustrates.

EXAMPLE 33 *Assume* $\mathcal{I}_{A_i} = \{\neg a \vee \neg b\}$ *and* $\mathcal{I} = \{a, b\}$. *Any reasonable contraction operator, cf. [13], would change* $\mathcal{I}_{A_i}$ *in a minimal way such that* $\mathcal{I}_{A_i} - \{\neg a, \neg b\} \nvdash \neg a$ *and* $\mathcal{I}_{A_i} - \{\neg a, \neg b\} \nvdash \neg b$. *In this case we get* $\mathcal{I}_{A_i} - \{\neg a, \neg b\} = \mathcal{I}_{A_i}$, *but obviously* $\mathcal{I}_{A_i} \cup \mathcal{I} \vdash \perp$.

5.3 Analysis

We first illustrate our approach with an example.

EXAMPLE 34 *Consider again Example 2 given in Section 1 where the agent* Sam *wants to go on vacation. Sam's boss* Bob *doesn't want Sam to go on vacation at this time of the year and tells him that he has to do some work. However, Sam is aware of the fact that* Paul*, a good colleague, can do his work. Now Paul becomes ill—and therefore cannot take Sam's duties—so Sam has to revise his beliefs accordingly.*

In this scenario let $\mathbb{A} = \{A_S, A_P, A_B, A_C\}$ *where* A_S *is Sam,* A_P *is an Sam's colleague Paul,* A_B *is an Sam's boss, and* A_C *is his assigned client Carl. Consider the sentences* v, w, r *and* i *with the following informal interpretations.*

v :	*Sam go on vacation*
w :	*There is work to do*
r :	*Paul can do Sam's work*
i :	*Paul is ill*

Now consider Sam's belief base $\mathcal{K}_{A_S}$ *given via* $\mathcal{I}_{A_S} = \{A_S{:}v, A_C{:}\neg w, A_P{:}r, A_P{:}r \Rightarrow v, A_S{:}\neg w \Rightarrow v\}$ *as was shown in Example 2 of Section 1. Furthermore, let the credibility order among agents according to Sam* ($<_{A_S}$) *be defined via* $A_S <_{A_S} A_P <_{A_S} A_B <_{A_S} A_C$.

As was introduced in Example 2 of Section 1, consider the new information $\Phi = \{A_B{:}w, A_P{:}i, A_B{:}i \Rightarrow \neg r\}$ *stemming from communication with Sam's boss. As one can see there are some arguments for and against* w, i *and* r *in* $\mathcal{I}_{A_S} \cup \Phi$, *e. g., arguments for and against* w *are* $\langle\{A_B{:}w\}, w\rangle$, $\langle\{A_C{:}\neg w\}, \neg w\rangle$.

We compute the argument structures $\Gamma_{\mathcal{I}_{A_S} \cup \Phi}(\alpha) = (\mathcal{P}, \mathcal{C})$ *for each sentence* $\alpha \in \mathsf{Form}(\Phi)$ *with respect to* $\mathcal{I}_{A_S} \cup \Phi$ *as follows.*

(w). *There is exactly one argument tree* τ_1 *for* w *and one argument tree* τ_2 *for* $\neg w$ *in* $\mathcal{I}_{A_S} \cup \Phi$. *In* τ_1 *the root is the argument* $\mathcal{A} = \langle\{A_B{:}w\}, w\rangle$ *which has the single canonical undercut* $\mathcal{B} = \langle\{A_C{:}\neg w\}, \neg w\rangle$. *In* τ_2

the situation is reversed and the root of τ_2 *is the argument* $\mathcal{B}$ *which has the single canonical undercut* $\mathcal{A}$*. Therefore, the argument structure for* w *is given via* $\Gamma_{\mathcal{I}_{A_S} \cup \Phi}(w) = (\{\tau_1\}, \{\tau_2\})$*. It follows that* $\gamma^c_{A_S}(\tau_1) = 0$*,* $\gamma^c_{A_S}(\tau_2) = 1$ *and* $\sum_{\tau \in \mathcal{P}} \gamma^c_{A_S}(\tau_1) - \sum_{\tau \in \mathcal{C}} \gamma^c_{A_S}(\tau_2) = -1$ *which means that* w *is* rejected*.*

(i)*. There is exactly one argument tree* τ_1 *for* i *and one argument tree* τ_2 *for* $\neg i$ *in* $\mathcal{I}_{A_S} \cup \Phi$*. In* τ_1 *the root is the argument* $\mathcal{A} = \langle \{A_P : i\}, i \rangle$ *which has the single canonical undercut* $\mathcal{B} = \langle \{A_B{:}i \Rightarrow \neg r, A_P{:}r\}, \neg i \rangle$*. In* τ_2 *the situation is reversed and the root of* τ_2 *is the argument* $\mathcal{B}$ *which has the single canonical undercut* $\mathcal{A}$*. Therefore, the argument structure for* i *is given via* $\Gamma_{\mathcal{I}_{A_S} \cup \Phi}(i) = (\{\tau_1\}, \{\tau_2\})$*. It follows that* $\gamma^c_{A_S}(\tau_1) = \gamma^c_{A_S}(\tau_2) = 0$ *and* $\sum_{\tau \in \mathcal{P}} \gamma^c_{A_S}(\tau_1) - \sum_{\tau \in \mathcal{C}} \gamma^c_{A_S}(\tau_2) = 0$ *which means that the status of* i *is* undecided*.*

$(i \Rightarrow \neg r)$*. There is exactly one argument tree* τ_1 *for* $i \Rightarrow \neg r$ *and one argument tree* τ_2 *for* $i \wedge r$ *in* $\mathcal{I}_{A_S} \cup \Phi$*. In* τ_1 *the root is the argument* $\mathcal{A} = \langle \{A_B : i \Rightarrow \neg r\}, i \Rightarrow \neg r \rangle$ *which has the single canonical undercut* $\mathcal{B} = \langle \{A_P : i, A_P : r\}, i \wedge r \rangle$*. In* τ_2 *the situation is reversed and the root of* τ_2 *is the argument* $\mathcal{B}$ *which has the single canonical undercut* $\mathcal{A}$*. Therefore, the argument structure for* $i \Rightarrow \neg r$ *is given via* $\Gamma_{\mathcal{I}_{A_S} \cup \Phi}(i \Rightarrow \neg r) = (\{\tau_1\}, \{\tau_2\})$*. It follows that* $\gamma^c_{A_S}(\tau_1) = 1$*,* $\gamma^c_{A_S}(\tau_2) = 0$ *and* $\sum_{\tau \in \mathcal{P}} \gamma^c_{A_S}(\tau_1) - \sum_{\tau \in \mathcal{C}} \gamma^c_{A_S}(\tau_2) = 1$ *which means that* $i \Rightarrow \neg r$ *is* accepted*.*

Due to the above evaluation the values of $C_{A_i}(\Phi)$ *and* $S_{A_i}(\Phi)$ *can be determined by*

$$S_{A_S}(\Phi) = \Phi \setminus \{A_B{:}w, A_P{:}i\} = \{A_B{:}i \Rightarrow \neg r\}$$
$$C_{A_S}(\Phi) = \Phi \setminus \{A_B{:}w\} = \{A_P{:}i, A_B{:}i \Rightarrow \neg r\}$$

If $*$ *is defined via (4) we obtain*

$$\begin{aligned} \mathcal{I}_{A_S} * S_{A_S}(\Phi) = \{&A_S{:}v, A_C{:}\neg w, A_P{:}r, A_P{:}r \Rightarrow v, \\ &A_S{:}\neg w \Rightarrow v, A_B{:}i \Rightarrow \neg r\} \\ \mathcal{I}_{A_S} * C_{A_S}(\Phi) = \{&A_S{:}v, A_C{:}\neg w, A_P{:}r \Rightarrow v, \\ &A_S{:}\neg w \Rightarrow v, A_P{:}i, A_B{:}i \Rightarrow \neg r\} \quad . \end{aligned}$$

The above example illustrates that our approach is quite complex and involves a sophisticated deliberation process for deciding how a non-prioritized revision should be performed. One might ask whether the argumentative decision process is necessary and if the same results could be obtained by a simpler approach that is based on direct comparisons of credibilities. The following definition of a transformation function is suitable to implement this idea.

DEFINITION 35 *Let* $\mathcal{K}_A = (\mathcal{I}_A, \leq_A)$ *and* $\mathcal{I} \subseteq \mathfrak{I}(\mathcal{L}(\mathsf{At}), \mathbb{A})$. *The function* H_A *is defined via*

$$H_A(\mathcal{I}) = \{A_i{:}\phi \in \mathcal{I} \mid \forall \langle \mathcal{I}', \neg\phi\rangle, \mathcal{I}' \subseteq \mathcal{I}_A \cup \mathcal{I}, \langle \mathcal{I}', \neg\phi\rangle \preceq_A \langle \{A_i{:}\phi\}, \phi\rangle\}.$$

In other words, the function H_A rejects an $A'{:}\phi \in \mathcal{I}$ if there is a proof for $\neg\phi$ in $\mathcal{I}_A \cup \mathcal{I}$ such that the least credible source of this proof is strictly more credible than A'. Therefore, this definition of a transformation function intuitively implements the idea of how a credibility-based revision should be defined. The question arises whether this definition of a transformation is sufficient for realizing a meaningful revision based on credibilities. In Example 36, we show that this is not the case.

EXAMPLE 36 *Let* $\mathbb{A} = \{A_1, A_2, A_3\}$ *be a set of agents and consider the belief base* $(\mathcal{I}_{A_1}, \leq_{A_1})$ *of agent* A_1 *given via*

$$\begin{aligned} \mathcal{I}_{A_1} &= \{A_3{:}b, A_3{:}a \Rightarrow \neg b, A_2{:}\neg c\} \\ \leq_{A_1} &= A_3 <_{A_1} A_2 <_{A_1} A_1 \quad . \end{aligned}$$

Assume now that A_1 *receives the new information* $\mathcal{I}$ *given via*

$$\mathcal{I} = \{A_3{:}a \Rightarrow c, A_3{:}a\}$$

and consider the revision of $\mathcal{I}_{A_1}$ *by* $\mathcal{I}$. *Observe that*

$$\begin{aligned} C_{A_1}(\mathcal{I}) &= \{A_3{:}a \Rightarrow c\} \\ H_{A_1}(\mathcal{I}) &= \{A_3{:}a \Rightarrow c, A_3{:}a\} = \mathcal{I} \end{aligned}$$

If $*$ *is defined via (4) we obtain*

$$\begin{aligned} \mathcal{I}_{A_1} * C_{A_1}(\mathcal{I}) &= \{A_3{:}b, A_3{:}a \Rightarrow \neg b, A_2{:}\neg c, A_3{:}a \Rightarrow c\} \\ \mathcal{I}_{A_1} * H_{A_1}(\mathcal{I}) &= \{A_3{:}b, A_3{:}a \Rightarrow c, A_3{:}a\} \quad . \end{aligned}$$

As one can see, the revision based on C_{A_1} *differs from the revision based on* H_{A_1} *which stems from* $A_3 : a \in H_{A_1}(\mathcal{I})$ *and* $A_3 : a \notin C_{A_1}(\mathcal{I})$. *The reason for* $A_3{:}a \in H_{A_1}(\mathcal{I})$ *is that there are two proofs for* $\neg a$ *in* $\mathcal{I}_{A_1} \cup \mathcal{I}$*—*$\{A_3 : b, A_3 : a \Rightarrow \neg b\}$ *and* $\{A_2 : \neg c, A_3 : a \Rightarrow c\}$*—and the credibility of the least credible agent in both proofs—which is* A_3*—is not strictly greater than the credibility of* $A_3{:}a$*—which is* A_3 *as well. Therefore,* H_{A_1} *accepts* $A_3{:}a$ *for revision. For* C_{A_1} *the situation is different. As* $\langle \{A_3{:}a\}, a\rangle$ *is the only* *argument for* a *and there are two arguments—*$\langle\{A_3:b, A_3:a \Rightarrow \neg b\}, \neg a\rangle$ *and* $\langle\{A_2:\neg c, A_3:a \Rightarrow c\}, \neg a\rangle$*—for* $\neg a$ *the argumentative evaluation of* a *results in the three argument trees depicted in Figure 2 and Figure 3. As all arguments appearing in the argument trees have the same least credible source* A_3 *no argument is ignored in the evaluation. Therefore the tree for argument* $\langle\{A_3:a\}, a\rangle$ *is categorized to* 0 *and both trees for* $\neg a$ *are categorized to* 1*. By (3) it follows that* $A_3:a$ *is not accepted for revision by* C_{A_1}*. An implication of this decision is that in* $\mathcal{I}_{A_1} * C_{A_1}(\mathcal{I})$ *the information* $A_2:\neg c$*—which is the single piece of information that comes from more credible information than any other piece of information—is retained.*

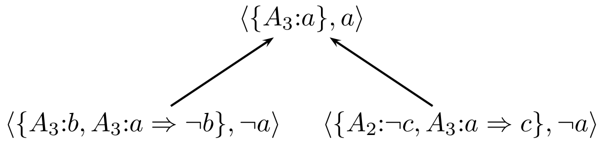

Figure 2. Argument tree in Example 36

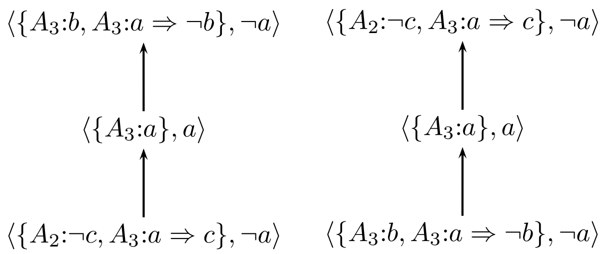

Figure 3. Argument trees in Example 36

As for formal properties for transformation functions and belief revision our approach behaves well. For the following results source annotations of formulas can be neglected.

PROPOSITION 37 *Let* A *be some agent. The transformation functions* S_A *and* C_A *satisfy* inclusion*,* weak inclusion*,* weak extensionality*,* consistency preservation*, and* weak maximality*.*

By exploiting Proposition 6, see also [29], we obtain the following result.

COROLLARY 38 *Let A be some agent. The operators $\circ^C_A$ and $\circ^S_A$ are non-prioritized multiple base revision operators.*

The above corollary shows that argumentative credibility-based revision conforms with expectations to non-prioritized revision.

6 Related Work

This chapter combines works on selected revision with deductive argumentation, and furthermore makes use of credibilities for revision in multiagent systems. It is based on previous work on credibility based multi-source belief revision as well as on the use of argumentation in multi-agent systems. More precisely, our approach to the model for multi-source belief revision has first been presented in [33, 34] and is combined with the selective revision operator as introduced in [29]. The base approach of Tamargo et al. is similar in its idea to the approaches of [9] and [6].

In [9], it is considered that agents detect and store in tables the *nogoods*, which are the minimally inconsistent subsets of their knowledge bases. A *good* is a subset of the knowledge base such that: it is not inconsistent (it is not a superset of a *nogood*), and if augmented with whatever else assumption in knowledge base, it becomes inconsistent. In contrast to our approach, they do not remove beliefs to avoid a contradiction, they choose which is the new preferred *good* among them in knowledge base. In [6], a *scenario* (set of incoming information) presented by a source is treated as a whole and not sentence by sentence, and therefore, it can be inconsistent. A relation of *trustworthiness* is introduced over sets of sources and not between single sources. Besides, if two sources give the same piece of information α, and a single agent gives $\neg\alpha$, then α will be preferred, that is, the decision is based on majority.

Selective revision is one of the most general non-prioritized revision operator of the type *decision+revision* [25]. Moreover it allows for partial acceptance of the input, in contrast to most other approaches. Apart from *decision+revision* approaches there are *expansion+consolidation* approaches to non-prioritized belief revision. These perform a simple expansion by the new information, i. e. $\mathcal{K} \cup \Phi$, and then apply a consolidation operator ! that restores consistency, i. e. $\mathcal{K} * \Phi = (\mathcal{K} \cup \Phi)!$. This approach is limited to belief bases since all inconsistent belief sets are equivalent, i. e. $Cn(\perp) = \mathcal{L}(\mathsf{At})$. An instantiation of such an operator that is similar to the setup used in this work has been presented in [11]. The input considered for revision consists of a set of sentences that form an explanation of some claim in the same form as the argument definition used here. However, as with all approaches of the type *expansion+consolidation*, new and old information are completely equivalent for consolidation. In contrast, the approach presented here makes use of two different mechanisms to first decide which part of the input and whether it shall be accepted and then perform prioritized belief revision of the old information.

While there has been some work on the revision of argumentation systems, very little work on the application of argumentation techniques for the revision process has been done so far, cf. [12]. In fact, the work most related to the work presented here makes use of negotiation techniques for belief revision [5, 36], without argumentation. In the general setup of [5] a symmetric merging of information from two sources is performed by means of a negotiation procedure that determines which source has to reduce its information in each round. The information to be given up is determined by another function. The negotiation ends when a consistent union of information is reached. While this can be seen as a one step process of merging or consolidation in general, the formalism also allows to differentiate between the information given up from the first source and the second source. In [5], this setting is then successively biased towards prioritizing the second source which leads to representation theorems for operations equivalent to selective revision satisfying *consistent expansion* and for classic AGM operators. However, the negotiation framework used in [5] is very different from the argumentation formalism used here and also very different from the setup of selective revision. Moreover, the functions for the negotiation and concession are left abstract.

In [36] mutual belief revision is considered where two agents revise their respective belief states by information of the other agent. Both agents agree in a negotiation on the information that is accepted by each agent. The revisions of the agents are split into a selection function and two iterated revision functions which leads to operators satisfying *consistent expansion*. The selection function is then a negotiation function on two belief sets that represent the belief sets that each agent is willing to accept from the other agent. This setting has a very different focus as ours and also does not specify the selection function.

There is also work on the use of argumentation to reason about trust, with [35] being the most recent work in this area. In [35] a meta-argumentation approach is used to not only argue by taking the trustworthiness of information sources into account while evaluating the acceptance of arguments, but also to argue about the trustworthiness itself. In these approaches it is determined for a given set of arguments from different sources which ones are accepted and which ones are not. Dynamics of the system in terms of belief revision are not considered. In contrast, here we treat a belief revision problem of non-argumentative belief bases by employing argumentation in the selection process of belief revision.

While the concepts of trust and reputation are complex, in this approach we have taken the position that they can be seen as a kind of credibility value that the agents assign to each other. In contrast to this work, in [31] a model for reputation is presented that takes into account the social dimension of agents and a hierarchical ontology structure. They show how the model relates to other systems and provide initial experimental results about the benefits of using a social view on the modeling of reputation.

7 Conclusions

In this chapter we showed how argumentation and selective revision can be combined to obtain a non-prioritized revision operator where the incoming information can be evaluated in order to decide if it is accepted or rejected. First, we defined a revision operator that only accepts new information if the information is justifiable with respect to an argumentative evaluation. Then, we went into the details of a multi-agent revision framework based on using deductive argumentation and credibilities for deciding whether new information should be accepted for revision. We used the very general framework of multiple belief base revision and investigated a scenario where an agent has to revise its belief base of propositional formulas with a set propositional formulas. Formulas are annotated with an agent identifier which defines the source of the information. The represent the credibility of each agent by a total preorder over all agent identifiers which carries over to the annotated formulas. We developed an argumentation procedure based on credibility that decides which formulas of the set should be accepted for (prioritized) revision. We investigated the properties of our approach and compared it to a simple approach for multi-agent revision and other related work.

Our approach is concerned with revising the actual content of the belief base of an agent given some *static* credibility assessment. That is, the credibilities of the agents in the system are fixed (subjectively) and must not change. However, this may not be the case in real-world scenarios, see [32] for a discussion. In particular, information received from an agent may change the subjective assessment of its credibility: if an agent often gives good arguments or his information is confirmed by more credible agents then this agent should be assessed to be more credible as well. The dynamics of credibility assessments can be approached by interpreting credibilities not as annotations but as formulas on the object level and to use traditional revision methods for them as well. Part of future work is on investigating dynamical credibility assessments within our framework of multi-agent revision.

Acknowledgements: This work is partially supported by PGI-UNS and

CONICET Argentina. Patrick Krümpelmann is supported by the DFG, Collaborative Research Center SFB 876, project A5,
`http://sfb876.tu-dortmund.de`.

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