A General Approach to Reasoning with Probabilities (Extended Abstract)

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Abstract

We propose a general scheme for adding probabilistic reasoning capabilities to any knowledge representation formalism.

1 Introduction

The ability to reason under uncertainty is a core requirement for most intelligent systems and many approaches for uncertain reasoning have been proposed in the area of *knowledge representation and reasoning* (KR) and *artificial intelligence* (AI) in general, see e. g. (Pearl 1988). In general, we can distinguish between *qualitative* uncertain reasoning and *quantitative* uncertain reasoning. The former encompasses approaches such as *default logic* (Reiter 1980), *answer set programming* (Gelfond and Lifschitz 1991), or *abstract argumentation* (Dung 1995). The latter makes use of formalisms such as *probability theory* (Pearl 1988), or *fuzzy logic* (Hájek 1998). A common approach to define a new quantitative model for uncertain reasoning is to take some non-qualitative approach—which may either be a qualitative model as mentioned before or something completely different such as propositional logic—add quantities to the syntax and define a new quantitative semantics on top of that. This approach is followed by e. g. probabilistic logics (Nilsson 1986; Halpern 1990); distribution semantics for logic programming (Sato 1995), then implemented in ProbLog (Raedt, Kimmig, and Toivonen 2007); P-log (Baral, Gelfond, and Rushton 2004); probabilistic approaches to formal argumentation (Li, Oren, and Norman 2011; Hunter 2012), any many more.

We aim at unifying many of the aforementioned approaches and define a general methodology for reasoning with quantitative uncertainty. This allows for a general study of its properties while abstracting away from any specific instantiation. We focus on probability theory as a means for quantitative uncertain reasoning but a similar methodology can be defined by building on other formalisms such as fuzzy logic or Dempster-Shafer theory. We start by considering an arbitrary *base logic* and define its *probabilistic augmentation* by extending the syntax to allow for annotated probabilities on each formula. Therefore, a knowledge base of probabilistic augmentation consists of a set of formulas, each annotated with a probability. We define a general probabilistic semantics on top of the built-in semantics of the base logic by (1) considering each subset of the knowledge base, (2) performing ordinary inference within the subset, and (3) accumulating the inferences by taking the probabilities into account. This gives us a general methodology for defining probabilistic versions of existing knowledge representation formalisms, and is inspired by many concrete realisations such as the distribution semantics for logic programming (Sato 1995).

2 Preliminaries

We consider a very general definition for a logic. For a set S let 2^S denote its power set.

Definition 1. *A* logic $\mathcal{L}$ *is a tuple* $\mathcal{L} = (\mathcal{W}, \mathcal{V}, \models)$ *where* $\mathcal{W}$ *is the set of well-formed formulas,* $\mathcal{V}$ *is the set of "inferrable" formulas, and* $\models \subseteq 2^{\mathcal{W}} \times \mathcal{V}$ *is an inference relation.*

As we aim to model a wide range of logics we explicitly distinguish between well-formed formulas $\mathcal{W}$ and formulas that can be inferred in the formalism $\mathcal{V}$.

We write $\mathcal{K} \models \phi$ ("$\mathcal{K}$ entails ϕ") instead of $(\mathcal{K}, \phi) \in \models$ for $\mathcal{K} \subseteq \mathcal{W}$, $\phi \in \mathcal{V}$. $\mathcal{K} \subseteq \mathcal{W}$ is $\models$-inconsistent if $\mathcal{K} \models \phi$ for all $\phi \in \mathcal{V}$; otherwise $\mathcal{K}$ is $\models$-consistent.

Example 1. *Let us consider answer set programming (Gelfond and Lifschitz 1991) and define* $\mathcal{L}_{ASP} = (\mathcal{W}_{ASP}, \mathcal{V}_{ASP}, \models_{ASP})$ *where* $\mathcal{W}_{ASP}$ *is the set of all extended logic program rules; and* $\mathcal{V}_{ASP}$ *is the set of all ground literals.*

For a set $P \subseteq \mathcal{W}_{ASP}$ *(also called answer set program) and a set* M *of ground literals the* reduct P^M *is defined via*

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$P^M = \{head(r) \leftarrow body^+(r) \mid r \in ground(P), \quad M \cap body^-(r) = \emptyset\}$.

A set M of ground literals is called answer set *if it is the minimal model of P^M. Then $P \models_{ASP} H$ for a ground literal H iff $H \in M$ for all answer sets M.*[1] *If there are no answer sets in P, we define $P \models_{ASP} \phi$ for all $\phi \in \mathcal{V}_{ASP}(\mathit{Pred}, U, V)$ (P is $\models_{ASP}$-inconsistent).*

3 Probabilistic Augmentations

Let $\mathcal{L} = (\mathcal{W}, \mathcal{V}, \models)$ be some logic, which will also be referred to as *base logic* in the following. We define its *probabilistic augmentation* $\mathcal{Z}(\mathcal{L}) = (\widehat{\mathcal{W}}, \widehat{\mathcal{V}}, \widehat{\models})$ as follows.

The languages $\widehat{\mathcal{W}}$ and $\widehat{\mathcal{V}}$ consist of the quantification of formulas of $\mathcal{L}$ with probabilities: $\widehat{\mathcal{W}} = \{\phi : p \mid \phi \in \mathcal{W}, p \in [0,1]\}$ and $\widehat{\mathcal{V}} = \{\phi : p \mid \phi \in \mathcal{V}, p \in [0,1]\}$. The semantics of $\mathcal{Z}(\mathcal{L})$ are defined in terms of probabilities of subsets of a knowledge base $\mathcal{K} \subseteq \widehat{\mathcal{W}}$. For every $\mathcal{K} \subseteq \widehat{\mathcal{W}}$ define $\mathcal{K}\downarrow = \{\phi \mid \phi : p \in \mathcal{K}\} \subseteq \mathcal{W}$. In other words, $\mathcal{K}\downarrow$ is the *flattened*—i.e. without probabilities—version of the knowledge base $\mathcal{K}$. We define now the *general probability* $P_\mathcal{K}$ of subsets of a probabilistic knowledge base $\mathcal{K} \subseteq \widehat{\mathcal{W}}$ via $P_\mathcal{K}(\mathcal{K}') = \prod_{\phi : p \in \mathcal{K}'} p \prod_{\phi : p \in \mathcal{K} \setminus \mathcal{K}'} (1-p)$ for all $\mathcal{K}' \subseteq \mathcal{K}$. Observe that $P_\mathcal{K}$ is indeed a probability distribution over subsets of $\mathcal{K}$.

Theorem 1. *For every $\mathcal{K} \subseteq \widehat{\mathcal{W}}$, $\sum_{\mathcal{K}' \subseteq \mathcal{K}} P_\mathcal{K}(\mathcal{K}') = 1$.*

Based on the general probability $P_\mathcal{K}$ we can define the *degree of belief* of any formula $\phi \in \mathcal{V}$ wrt. $\mathcal{K}$ via $\Pi_\mathcal{K}(\phi) = \sum_{\mathcal{K}' \subseteq \mathcal{K}, \mathcal{K}'\downarrow \models \phi} P_\mathcal{K}(\mathcal{K}')$.

In other words, a probabilistic knowledge base $\mathcal{K} \subseteq \widehat{\mathcal{W}}$ defines a probability distribution over all subsets of $\mathcal{K}$. For each subset $\mathcal{K}' \subseteq \mathcal{K}$, we consider its flattened version $\mathcal{K}'\downarrow$ and decide using the base logic $\mathcal{L}$, whether $\mathcal{K}'\downarrow$ infers ϕ. We sum up the probabilities of all subsets where this is the case in order to obtain the degree of belief of ϕ wrt. the probabilistic knowledge base $\mathcal{K}$.

Based on $\Pi_\mathcal{K}$ we define probabilistic inference $\widehat{\models}$ via $\mathcal{K} \widehat{\models} \phi : p$ if $\Pi_\mathcal{K}(\phi) = p$ for all $\mathcal{K} \subseteq \widehat{\mathcal{W}}$.

Let us consider the probabilistic augmentation of an answer set program $\mathcal{Z}(\mathcal{L}_{\mathsf{ASP}})$ and let us illustrate its semantics by the means of an example.[2]

Example 2. *Consider the following answer set program augmented with probabilities:*

$$\begin{array}{ll} \mathit{drill} \leftarrow \mathit{alarm}, \mathit{not\ real} & : 0.2 \\ \mathit{real} \leftarrow \mathit{alarm}, \mathit{not\ drill} & : 0.9 \\ \mathit{alarm} & : 1 \end{array}$$

Hence, $\mathcal{K} \widehat{\models_{ASP}} \mathit{real} : 0.72$ as the only subset $\mathcal{K}' \models_{ASP} \mathit{real}$ is $\mathcal{K}' = \{r \leftarrow a, \mathit{not}\ d; a\}$.

4 Conclusion

We developed a general scheme for adding probabilistic reasoning capabilities to any knowledge representation formalism. Pivotal in our proposal is the notion of probabilistic augmentation of a knowledge representation formalism, which extends it by enabling probabilities to be expressed on the logical formulas of the chosen formalism.

This work lays the foundation for a general approach to probabilistic reasoning that has the potential to create synergies between different fields interested in incorporating probability into a specific framework. Our logical setting is general enough to capture a wide variety of logics as a base logic such as classical logic, modal logic, and even abstract argumentation.

[1] $\models_{\mathsf{ASP}}$ can also be defined credulously by requiring that H is contained in *some* answer set.

[2] Although no formal definitions are provided, at first sight (Dragiev et al. 2016) might seem close in spirit to our proposal.

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